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ABSTRACT

This paper clarifies the relationship between the absorbed crush energy and the dissipated crush energy and explores the use of each in crush and conservation of energy analysis. There is inconsistency and confusion in the literature of accident reconstruction regarding when crush analysis and conservation of energy analysis should use the *absorbed* crush energy and when it should use the *dissipated* crush energy. It is demonstrated in this paper that crush analysis calls for the absorbed energy, while conservation of energy analysis calls for the dissipated energy. However, this paper also shows that the equations of crush analysis and conservation of energy analysis can be written in terms of *either* the absorbed or the dissipated crush energies, since the absorbed and dissipated energies are related through the coefficient of restitution (when friction-type energy losses are assumed negligible). The assumptions of crush analysis are explored in order to develop a consistent approach.

CLARIFYING THE USE OF THE ABSORBED AND DISSIPATED ENERGIES

Equations (1) and (2) are the well-known crush analysis equations that relate the crush energy to the approach velocity and the vehicle changes in velocity for a central impact [8].

$$V_A = \sqrt{\frac{M_1 + M_2}{M_1 M_2} \cdot 2 \cdot E} \qquad (1)$$

$$\Delta V_i = \frac{1}{M_i} \sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot E} \qquad (2)$$

In Equations (1) and (2), V_A is the relative approach velocity at impact, ΔV_i is the approach phase velocity change for the vehicle under consideration (i = 1,2), M_1 and M_2 are the vehicle masses, and E is the total crush energy for the impact. Equation (2) does not account for the ΔV that occurs during the restitution phase of the impact.

There is inconsistency and confusion in the literature of accident reconstruction regarding whether Equations (1) and (2) should employ the *absorbed* crush energy E_A or the *dissipated* crush energy E_d.[1,2,3,4] The absorbed energy is defined as the system deformation energy at the point of maximum dynamic crush. Prior to the end of the impact, the structure rebounds partially and restores some of the absorbed deformation energy back to the vehicles in the form of kinetic energy. The system deformation energy at the point the vehicles separate, after the partial restoration of energy, is the dissipated energy.[5]

To clarify the relationship of the absorbed crush energy to the dissipated crush energy and to explore the use of each in Equations (1) and (2), consider the physics of a barrier impact crash test. During a barrier impact,

[1] In Reference 3, Carpenter and Welcher observed, "Review of the literature related to collision energy analysis reveals numerous works with conflicting and, in some cases, incorrect usage of the terms energy absorption, energy recovery and energy dissipation." In their paper, they clarify the difference between the absorbed and dissipated energies in a manner consistent with the discussion contained in this paper. However, Carpenter and Welcher continue, "The residual deformation to a vehicle only correlates with the dissipated energy, not the absorbed energy. Traditional accident reconstruction methods utilizing energy and post-crash crush data are analyzing the relationship between the dissipated energy and the BEV or Delta-V, not the absorbed energy." As will be discussed below, it is actually the absorbed energy that correlates to the ΔV and the BEV.

[2] In Reference 7, Happer, et al, cite an equation identical to Equation (25) below, with the exception that they use the absorbed energy instead of the dissipated energy. It will be shown below that the role of the coefficient of restitution in Equation (25) is to convert the dissipated energy into the absorbed energy, and so, it is actually the dissipated energy that should be used in this equation.

[3] Reference 6 also confuses the use of the terms *absorbed energy* and *dissipated energy*. For instance, on Page 70-16, the narrative reads as follows: "The G term can be thought of as the energy dissipated without permanent damage." As the discussion that follows will show, this statement should have been given in terms of the absorbed energy, not the dissipated energy.

[4] There is nothing unique about the examples given in the first three footnotes. The intent of these footnotes is simply to offer representative examples, not to single out these specific publications as unusual.

[5] This statement is true when friction-type energy losses are assumed negligible. The derivation of the crush analysis equations [Equations (1) and (2)] makes this assumption and it is employed throughout this paper.

crushing of the vehicle structure absorbs the vehicle's initial kinetic energy. This absorption of energy occurs during the approach phase of the impact as the vehicle structure crushes to its maximum depth. This maximum crush depth occurs when the vehicle velocity goes to zero, the point of common velocity with the barrier. Theoretically, the maximum force occurs at the same time that the crush reaches its maximum depth.

After the approach phase is complete, the impact force drops quickly as the vehicle structure experiences a partial rebound from the maximum dynamic crush. This structural rebound has the effect of imparting a velocity to the vehicle, in the opposite direction of its initial velocity, and thus, of restoring some kinetic energy to the vehicle. When the vehicle separates from the barrier, the collision force goes to zero and the vehicle structure finishes rebounding to its final residual crush. The phase during which the vehicle rebounds from the barrier and experiences partial structural restoration is referred to as the restitution phase of the collision.

The force-dynamic crush curve from a barrier impact can be idealized as shown in Figure 1 [4, 10, 11].

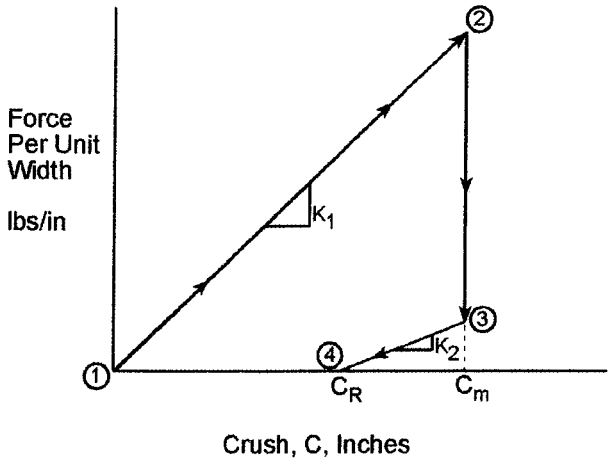

Figure 1 – Dynamic Force-Crush Curve

This force-crush curve has four points that define the structural response of the vehicle. The curve begins at Point 1, where there is zero crush and zero force. At Point 2, the curve has reached the maximum dynamic crush value C_m and a maximum force value. Between Points 2 and 3, the curve drops to a lower force level. Then, from Point 3 to Point 4, the structure experiences partial restoration to the residual crush C_R and the collision force returns to zero.[6]

There are three energy values that can be identified using the force-crush curve of Figure 1. The area underneath the line from Point 1 to Point 2 is equal to the *absorbed* energy. The area underneath the line from Point 3 to Point 4 is referred to as the *restored* energy. The difference between the absorbed and the restored energies is the *dissipated* energy (energy loss). Figure 2 shows each of these areas graphically.

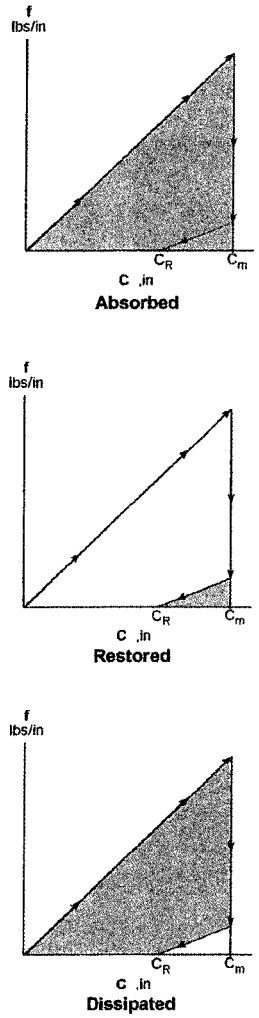

Figure 2 – Absorbed, Restored, and Dissipated Energies

Review of the derivation of Equations (1) and (2) reveals that it is the absorbed energy that should be employed in these equations. While the derivation of Equations (1) and (2) appeared in the literature in 1975, the contours of the derivation are repeated here so that its key features can be highlighted. Equations (1) and (2) are based on the one-dimensional, two degree-of-freedom mass-spring model shown in Figure 3. In this model, the masses represent the non-deforming region of each vehicle and the springs represent the deforming region of each vehicle.

The equations of motion for the masses of Figure 3 can be written in the following form:

[6] Other authors have proposed different force-crush curves (Reference 13). The discussion in this paper is applicable regardless of which idealized force-crush shape one chooses to use.

$$M_1\ddot{X}_1 = \left(\frac{K_1K_2}{K_1+K_2}\right)\cdot(X_2 - X_1) \tag{3}$$

$$M_2\ddot{X}_2 = \left(\frac{K_1K_2}{K_1+K_2}\right)\cdot(X_1 - X_2) \tag{4}$$

In Equations (3) and (4), M_1 and M_2 are the vehicle masses, X_1 and X_2 are the X-direction displacements of the masses from their original positions, $\ddot{X}_1$ and $\ddot{X}_2$ are the X-direction accelerations of the masses, and K_1 and K_2 are the stiffnesses of the springs.

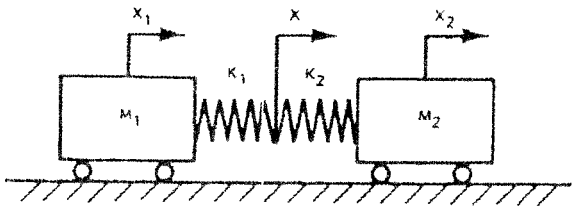

Figure 3 – The CRASH Impact Model[7]

Subtracting Equation (4) from Equation (3) and letting $\delta = X_1 - X_2$ yields Equation (5).

$$\ddot{\delta} + \Omega^2\delta = 0 \tag{5}$$

In Equation (5), ...

$$\Omega = \sqrt{\frac{M_1+M_2}{M_1M_2}\cdot\frac{K_1K_2}{K_1+K_2}} \tag{6}$$

At t = 0, the springs are uncompressed ($\delta = 0$) and the relative velocity of the masses is equal to $\dot{X}_{10} - \dot{X}_{20}$. Given these initial conditions, the solution to Equation (5) is given by Equation (7).

$$\delta = \left(\dot{X}_{10} - \dot{X}_{20}\right)\cdot\frac{1}{\Omega}\cdot\sin\Omega t \tag{7}$$

The maximum relative displacement of the vehicles is given by Equation (8) and occurs when the sine term in Equation (7) is equal to 1. This is the same time that the relative velocity between the masses goes to zero, *the time of common velocity.*

$$\delta_{max} = \left(\dot{X}_{10} - \dot{X}_{20}\right)\cdot\frac{1}{\Omega}. \tag{8}$$

[7] Figure 3 is reproduced from Reference 9.

Now let $\delta_1 = X_1 - X$ and $\delta_2 = X - X_2$. For force equilibrium it is necessary that

$$K_1\delta_1 = K_2\delta_2. \tag{9}$$

And since $\delta = \delta_1 + \delta_2$, we can write

$$\delta_1 = \left(\frac{K_2}{K_1+K_2}\right)\cdot\delta. \tag{10}$$

Using Equation (9), (10) and $\delta = \delta_1 + \delta_2$, Equation (8) can be written as

$$\dot{X}_{10} - \dot{X}_{20} = \sqrt{\left(\frac{M_1+M_2}{M_1M_2}\right)\cdot\left(K_1\delta_{1,max}^2 + K_2\delta_{2,max}^2\right)} \tag{11}$$

The energy *absorbed* in compressing each of the springs can be expressed as

$$E_{A,i} = \frac{1}{2}K_i\delta_{max,i}^2. \tag{12}$$

Equation (12) is the *absorbed* energy because it occurs at the time of maximum crush, prior to any rebound of the spring that represents the vehicle structure. Therefore, Equation (11) can be written as

$$V_A = \dot{X}_{10} - \dot{X}_{20} = \sqrt{\frac{M_1+M_2}{M_1M_2}\cdot 2\cdot E_A} \tag{13}$$

In Equation (13), E_A is the total absorbed energy for the two-vehicle system. Equation (13) relates the total *absorbed* crush energy to the initial relative velocity, V_A, and is equivalent to Equation (1).

At the time of maximum mutual spring compression, the masses reach a common velocity, V_C. From conservation of momentum, this common velocity can be written as

$$V_C = \frac{M_1\dot{X}_{10} + M_2\dot{X}_{20}}{M_1+M_2}. \tag{14}$$

The changes in velocity experienced by the masses, ΔV_1 and ΔV_2, from the initial time, t=0, to the time of maximum mutual spring compression are, thus, given by Equations (15) and (16).

$$\Delta V_1 = \dot{X}_{10} - V_C = \dot{X}_{10} - \left(\frac{M_1\dot{X}_{10} + M_2\dot{X}_{20}}{M_1+M_2}\right) \tag{15}$$

$$\Delta V_2 = V_C - \dot{X}_{20} = \left(\frac{M_1 \dot{X}_{10} + M_2 \dot{X}_{20}}{M_1 + M_2} \right) - \dot{X}_{20} \qquad (16)$$

It is important to note that the velocity changes of Equations (15) and (16) are the velocity changes for the approach phase of the collision only. They do not include the restitution phase of the impact, during which there is an additional ΔV for each vehicle. Incorporating Equation (13) into Equations (15) and (16) and simplifying yields the following expression:

$$\Delta V_i = \frac{1}{M_i} \sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot E_A} \qquad (17)$$

Equation (17) relates the energy *absorbed* in crushing the vehicle structure to the change in velocity experienced by each vehicle during the approach phase of the impact. It is equivalent to Equation (2).

There are two important points to observe regarding the derivation of Equations (1) and (2). First, the model of Figure 3 only considers energy absorption due to physical displacement of the vehicle structure, as modeled by the springs. Energy loss due to intervehicular sliding (friction-type energy loss) is not considered. Thus, Equations (1) and (2) are applicable to impacts where the dominant mechanism of energy loss is vehicle deformation and where intervehicular sliding is negligible. Equations (1) and (2) do not hold when energy loss due to intervehicular sliding is considered. In cases where friction-type energy losses are significant and need to be considered, Equations (1) and (2) should not be applied without modification. In these cases, a direct application of the principle of impulse and momentum is more appropriate, as discussed by Brach in Reference 1.

The second observation that should made is that the derivation of Equations (1) and (2) relies on identifying the point at which the maximum dynamic crush is achieved, the point at which the vehicle reaches a common velocity with the barrier or the other vehicle and the point of maximum energy *absorption* (Point 2 in Figure 1). Confusion may arise surrounding this point since crush analysis uses measurement of the residual (static) crush to quantify the crush energy, while the maximum energy absorption occurs at the maximum dynamic crush depth. It is natural to think of the *residual* crush being associated with the dissipated energy, as it is in Figure 2. However, it should be kept in mind that the derivation of Equations (1) and (2) is separate from the derivation of the residual crush model that actually provides the crush energy estimate. Equations (1) and (2) simply provide the physical relationships between the crush energy and the approach velocity and velocity changes. They do not provide the tool for actually quantifying that crush energy.

Equation (18) is used in crush analysis to obtain the energy value for use with Equations (1) and (2). (16)

$$E = \left(\frac{B}{2} C_R^2 + A C_R + \frac{A^2}{2B} \right) \cdot w_0 \qquad (18)$$

In Equation (18), A and B are the crush stiffness coefficients and w_0 is the damage width. The A and B stiffness coefficients can be calculated such that the residual crush measurements yield *either* the absorbed or the dissipated crush energy. This being the case, the use of residual crush measurements in the model does not automatically imply that Equation (18) will yield the dissipated energy.

It can be further demonstrated that the absorbed energy should be used in Equations (1) and (2) by writing Equation 1 in a form appropriate for a barrier impact, where the mass of the barrier approaches infinity, as follows:

$$V_A = \sqrt{\frac{2 \cdot E_A}{M_1}} \qquad (19)$$

Equation (19) can be rewritten in the following form:

$$E_A = \frac{1}{2} M_1 V_A^2 \qquad (20)$$

For the barrier impact case, Equation (20) is true by definition. However, Equation (20) would not be true if the left hand side were written as the dissipated energy, since the dissipated crush energy is instead defined as follows:

$$E_d = \frac{1}{2} M_1 V_A^2 - \frac{1}{2} M_1 V_S^2 \qquad (21)$$

In Equation (21), E_d is the dissipated energy and V_S is the maximum velocity at which the vehicle separates from the barrier.

Equations (1) and (2), therefore, properly employ the absorbed crush energy. These equations can, however, be rewritten in a form that uses the dissipated energy, since the absorbed and dissipated energies are related through the coefficient of restitution (when friction-type energy losses are negligible). This is discussed in the next section.

REWRITTING EQUATIONS (1) AND (2)

To obtain a different form of Equations (1) and (2), note the following relationships between the absorbed, restored, and dissipated energies:

$$E_d = E_A - E_R \tag{22}$$

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} \tag{23}$$

Equation (22) is true by definition when friction-type energy losses are negligible and follows from the geometry of Figure 2. Equation (23) defines the coefficient of restitution in terms of the restored and absorbed energies for crush analysis. When combined with the effective mass concept [14], Equation (23) can be shown to be applicable to a general planar impact during which friction-type energy losses are negligible. Reference 15 contains a derivation of Equation (23) for such a general planar impact. When combined, Equations (22) and (23) yield the following relationship between the absorbed and dissipated energies:

$$E_A = \frac{E_d}{1-\varepsilon^2} \tag{24}$$

Substitution of Equation (24) into Equations (1) and (2) yields the following set of equations:

$$V_A = \sqrt{\frac{M_1 + M_2}{M_1 M_2} \cdot 2 \cdot \frac{E_d}{1-\varepsilon^2}} \tag{25}$$

$$\Delta V_i = \frac{1}{M_i}\sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot \frac{E_d}{1-\varepsilon^2}} \tag{26}$$

Thus, while the derivation of Equations (1) and (2) dictates that the absorbed energy be input into the equations, the coefficient of restitution can be used to write a form of these equations that uses the dissipated energy. Within Equations (25) and (26), the coefficient of restitution converts the dissipated energy into the absorbed energy. *It is important to note that even though Equation 26 includes the coefficient of restitution, it does not account for the restitution phase of the impact and still only yields the approach phase velocity changes.*

For severe impacts, confusing the absorbed and dissipated energies will have little effect on the calculated closing speed and ΔVs. For severe frontal impacts, where the coefficient of restitution will usually fall between 0.1 and 0.2, the difference between the absorbed and dissipated energies will only be between 1 and 4 percent. Since this difference will fall underneath the square root signs of the equations for closing speed and ΔV, the difference in calculated values that will result from confusing the absorbed and dissipated energies will be negligible. The difference between the absorbed and restored energies would, of course, become more significant for low speed impacts. However, the significance of the difference between the absorbed and restored energies can be considered negligible from an accuracy standpoint for more severe impacts and should not be over emphasized for these cases.

The primary goal of the discussion in this paper is not, therefore, to achieve any significant improvement in the accuracy of crush analysis. Instead, this paper aims at a clear and consistent theoretical description of the use of the absorbed and dissipated energies. The value of this discussion lies primarily in gaining understanding and clarity. Such understanding and clarity becomes essential when, for instance, one attempts to understand the relationship between the equations of crush analysis and the equations of planar impact mechanics – a discussion taken up later in this paper. Beyond that, there is always value in understanding the applications and limitations of the models that one employs.

GENERALIZING CRUSH ANALYSIS

The model of Figure 3 is one-dimensional, as are the equations that it yields, Equations (1) and (2) or Equations (25) and (26). Each vehicle is allowed to translate along a single coordinate direction and rotation is not considered. The restitution phase of the impact is considered negligible and since residual crush measurements are taken perpendicular to the original vehicle side, the crush energy calculated with Equation (18) inherently assumes that the collision force acted perpendicular to the original shape of the damaged side. Equations (1) and (2) and Equations (25) and (26) are, thus, applicable to central impacts where the restored energy is negligible and the collision force acts perpendicular to the original shape of the damaged vehicle side.

How is it that the one-dimensional equations of crush analysis can be extended to the general two-dimensional (planar) impact, where each vehicle has three degrees-of-freedom – two in translation and one in rotation – where the collision forces do not act through the vehicle centers of gravity, where the collision forces are not applied perpendicular to original shape of the damaged vehicle side, and where restitution is not negligible? This general application of the crush analysis equations is accomplished, first, by incorporating the restitution phase of the impact, second, by using the *effective mass concept*, and third, by adjusting the calculated damage energy to reflect non-perpendicular collision forces.

RESTITUTION

References 10, 11 and 15 discuss the incorporation of restitution into crush analysis and that discussion will not be taken up here. Suffice it to say here that if the coefficient of restitution is incorporated into the crush

analysis equations, then Equation (26) can be rewritten as follows:

$$\Delta V_i = \frac{1}{M_i}\sqrt{\frac{M_1 M_2}{M_1 + M_2}\cdot 2\cdot\frac{1+\varepsilon}{1-\varepsilon}\cdot E_d} \qquad (27)$$

For reasons that will be explained, Equation (27) has been written in a form that uses the *dissipated* crush energy.

EFFECTIVE MASS

The effective mass concept is discussed in Reference 14 and a complete description of the concept will not be taken up here. Suffice it to say here that the effective mass concept shifts the ΔV calculation to the point of collision force transfer using a calculation of the portion of the mass of each vehicle, the "effective mass," acting at that point of collision force transfer. The effective masses are given by Equation (28).

$$M_{e,i} = \frac{k_i^2}{k_i^2 + h_i^2} M_i = \gamma_i M_i \qquad (28)$$

In Equation (28) k_i represents the radius of gyration and h_i represents the collision force moment arm. When the effective masses are substituted into Equation (27), the resulting equation will yield the change in velocity for each vehicle *at the point of collision force transfer*, $\Delta V_i'$. The effective mass multiplier, γ_i, can then be used again to transfer the calculated ΔV back to the vehicle centers of gravity. The resulting equation is the center of gravity velocity changes for non-central impacts and is given as follows:

$$\Delta V_i = \frac{1}{M_i}\sqrt{2\overline{M_e}\cdot\frac{1+\varepsilon}{1-\varepsilon}\cdot E_d} \qquad (29)$$

The form of Equation (29) has been simplified using the following formula:

$$\overline{M_e} = \frac{\gamma_1 M_1 \gamma_2 M_2}{\gamma_1 M_1 + \gamma_2 M_2} \qquad (30)$$

NON-PERPENDICULAR COLLISION FORCES

Since residual crush measurements are taken perpendicular to the original shape of the damaged vehicle side, the damage energy that results from Equation (18) only yields the crush energy normal to the damaged vehicle side. The damage energy, thus, needs to be adjusted to include the portion of the collision force that acted tangential to the damaged vehicle side.

There are two basic approaches that have been proposed for adjusting the damage energy to reflect the tangential component of the collision force. The first approach is to adjust the residual crush depth so that it represents the distance the structure was displaced along the direction of the collision force. It is important to note that the normal and tangential directions relative to the vehicle are not necessarily the same as the normal and tangential directions for the overall impact, as they are used in planar impact mechanics [1]. Recall that Equations (1) and (2) are derived assuming that friction-type energy losses are negligible. If this is the case, there will not be any energy loss along the tangent direction for the overall impact, but only along the overall impact normal direction. Thus, when a non-perpendicular collision force is specified for crush analysis, this is equivalent to orienting the overall impact normal direction relative to the vehicle side normal. The crush depth is therefore adjusted to reflect its length along the collision normal. This approach is used in CRASH3 and it results in the energy being multiplied by the following multiplier [12]:

$$1 + \tan^2\alpha \qquad (31)$$

In Equation (31), α is the angle between the damaged vehicle side normal and the actual direction of the collision force.

References 5 and 17 have proposed that the width of the damaged region should also be adjusted to reflect the direction of the collision force. When both the crush depth and the crush width are adjusted, the crush energy multiplier that results is given by Equation (32).

$$\frac{1}{\cos\alpha} \qquad (32)$$

What is important to realize here is that, while Equations (31) and (32) clearly yield different numerical values, they share the same basic philosophy of adjusting the crush depth to reflect the distance the vehicle structure was displaced along the direction of the collision force (the impact normal direction). In this approach, there is no attempt to account for friction-type losses along the tangential direction and the dominant mechanism of energy loss, along the PDOF, is still considered to be physical displacement of the vehicle structure.

The second approach that has been proposed for adjusting the crush energy attempts to allow for friction-type losses along the tangent by introducing an intervehicular friction coefficient μ. This approach has resulted in the proposal of the following two additional energy multipliers [10, 12]:

$$(\cos\alpha+\mu\sin\alpha)^2 \tag{33}$$

$$1+\mu\tan\alpha \tag{34}$$

The problem with attempting to adjust the damage energy to include friction-type losses is that Equations (1), (2), (25), (26), and (29) are no longer valid once friction losses are considered. In Reference 1, Brach gives a more general form of Equations (29) that includes friction-type losses, as follows:[8]

$$\Delta V_i=\frac{1}{M_i}\sqrt{\frac{2\overline{M_e}(1+\mu^2)(1+\varepsilon)^2E_d}{(1-\varepsilon^2)+r^2\left[2\left(\frac{\mu}{\mu_{\max}}\right)-\left(\frac{\mu}{\mu_{\max}}\right)^2\right]}} \tag{35}$$

The dissipated energy for Equation (35) is given in Reference 1 by Equation (36), as follows:

$$E_d=\frac{1}{2}\overline{M_e}V_A^2\left\{(1-\varepsilon^2)+r^2\left[2\left(\frac{\mu}{\mu_{\max}}\right)-\left(\frac{\mu}{\mu_{\max}}\right)^2\right]\right\} \tag{36}$$

Incorporating the effective mass multipliers into Equation (25) and rewriting yields the following equation for the dissipated crush energy within CRASH-type crush analysis:

$$E_d=\frac{1}{2}\overline{M_e}V_A^2(1-\varepsilon^2) \tag{37}$$

Comparison of Equations (36) and (37) reveals that the dissipated energy in crush analysis does not include the friction-type losses, since Equation (37) does not contain either of the terms in Equation (36) that incorporate the intervehicular friction coefficient, μ. Even if Equation (37) could be adjusted using a multiplier to add in these friction losses, that would not be enough. Comparison of Equations (29) and (35) reveals that the equations of crush analysis do not only neglect friction losses in the calculation of the damage energy, they also neglect friction losses when relating the damage energy to ΔVs. In other words, both Equations (29) and (37) neglect the friction-type losses, and so, it is not sufficient in crush analysis to simply correct the calculated crush energy to include friction losses. Equation (29) would also have to be adjusted.

This leads to two important conclusions. First, the equations of crush analysis should not be applied in cases where friction-type energy loss is significant relative to the energy loss due to plastic deformation. Second, the first approach to adjusting the damage energy – adjusting the crush depth to reflect the direction of the collision force – is more consistent with the assumptions of crush analysis. In this approach, the one-dimensional Equations (1) and (2) are applied to analyze a general planar impact by first shifting the problem to the point of collision force transfer (effective mass concept), then orienting the problem along the direction of the collision force (crush depth adjustment), then calculating the ΔVs due to plastic deformation along the direction of the collision force at the point of collision force transfer, and finally shifting the solution back to the vehicle centers of gravity (effective mass concept). The effective mass concept and the adjustment of the residual crush depth to reflect the direction of the collision force are effectively used to reduce any general planar impact to a one-dimensional, central impact, with two bodies of some effective mass.

CONSERVATION OF ENERGY

Reference 14 described a method for combining crush analysis with conservation of energy analysis. In that discussion, the energy balance equation was written with the absorbed crush energies, since restitution was neglected in the derivation. However, when restitution is considered, the energy balance equation should be written with the dissipated energies as follows:

$$\begin{aligned}&\frac{1}{2}M_1V_1^2+\frac{1}{2}M_2V_2^2=\frac{1}{2}M_1V_{1f}^2+\frac{1}{2}M_2V_{2f}^2\\&+\frac{1}{2}I_1\dot{\psi}_{1f}^2+\frac{1}{2}I_2\dot{\psi}_{2f}^2+E_{d1}+E_{d2}\end{aligned} \tag{38}$$

In Equation (38), V_1 and V_2 are the vehicle impact velocities, V_{1f} and V_{2f} are the vehicle velocities at separation, $\dot{\psi}_{1f}$ and $\dot{\psi}_{2f}$ are the yaw velocities of the vehicles at separation, and E_{d1} and E_{d2} are the dissipated energies for the two vehicles. The initial rotational velocities are assumed negligible.

In this case, the dissipated energies are used because the point of reference in Equation (38) is no longer the point of common velocity during the impact, but the difference between the kinetic energy of the vehicles before and after the impact, including the restitution phase of the impact. The difference in kinetic energy immediately before and immediately after the impact is, by definition, the total dissipated energy.

So it is the point of reference used by each method that determines which crush energy quantity should be used. Equations (39) and (40) demonstrate this with the use of a shorthand form of the energy balance equation.

$$KE_i-KE_{cv}=E_A \tag{39}$$

$$KE_i-KE_{sep}=E_d \tag{40}$$

[8] In Reference 1, Brach limits the applicability of Equation (35) to particle impact collisions. However, through use of the effective masses, the equation can be applied to the general planar impact problem.

In Equations (39) and (40), KE_i denotes the kinetic energy of the two-vehicle system at the beginning of the impact, KE_{cv} is the kinetic energy of the system when the common velocity is reached on the contact surface between the vehicles, and KE_{sep} is the kinetic energy of the system when the vehicles separate. Equations (39) and (40), thus, define the principles necessary to achieve consistent use of the absorbed and dissipated energies. It should be noted that we have continued to neglect friction-type losses in this discussion. This is so that the resulting energy balance equations can be consistently applied within crush analysis.

From the equations in References 9 and 10 it can be shown that

$$\dot{\psi}_{1f} = \frac{h_1}{k_1^2}\Delta V_{A1}(1+\varepsilon) \tag{41}$$

In Equation (41), h_1 is the collision force moment arm about the vehicle center of gravity, k_1 is the radius of gyration, ε is the overall coefficient of restitution for the impact, and ΔV_{A1} is the approach phase velocity change.

Combining Equation (41) with Equation (2), written for the non-central impact case, yields Equation (42).

$$\dot{\psi}_{1f} = \frac{h_1}{k_1^2}\cdot(1+\varepsilon)\cdot\frac{1}{M_1}\sqrt{2\cdot\overline{M}_e\cdot E_A} \tag{42}$$

Substitution of Equation (42), along with its counterpart for Vehicle #2, into Equation (38) yields Equation (43).

$$\begin{aligned}&\frac{1}{2}M_1V_1^2+\frac{1}{2}M_2V_2^2=\\&\frac{1}{2}M_1V_{1f}^2+\frac{1}{2}M_2V_{2f}^2+\xi+E_{d1}+E_{d2}\end{aligned} \tag{43}$$

The ξ term in Equation (43) is given by Equation (44).

$$\xi = (1+\varepsilon)^2\cdot\beta\cdot\left(\frac{E_{d1}}{1-\varepsilon_1^2}+\frac{E_{d2}}{1-\varepsilon_2^2}\right) \tag{44}$$

In Equation (44), β is given by Equation (45).

$$\begin{aligned}\beta = &\frac{h_1^2}{k_1^2+h_1^2}\cdot\frac{\gamma_2 M_2}{\gamma_1 M_1+\gamma_2 M_2}+\\&\frac{h_2^2}{k_2^2+h_2^2}\cdot\frac{\gamma_1 M_1}{\gamma_1 M_1+\gamma_2 M_2}\end{aligned} \tag{45}$$

In Equation (44), ε_1 and ε_2 are vehicle-specific coefficients of restitution described in Reference 14.

In order to maintain the use of dissipated energy in Equation (43), Equation (44) utilized Equation (24). Now, to obtain consistency with Equations (1) and (2), Equation (24) can be substituted in into Equations (43) and (44) for each of the dissipated energy terms to yield Equations (46) and (47).

$$\begin{aligned}&\frac{1}{2}M_1V_1^2+\frac{1}{2}M_2V_2^2=\\&\frac{1}{2}M_1V_{1f}^2+\frac{1}{2}M_2V_{2f}^2+\xi+\\&(1-\varepsilon_1^2)E_{A1}+(1-\varepsilon_2^2)E_{A2}\end{aligned} \tag{46}$$

$$\xi = (1+\varepsilon)^2\cdot\beta\cdot E_A \tag{47}$$

DISCUSSION AND CONCLUSIONS

Determining whether accident reconstruction analysis should use the absorbed or the dissipated crush energy depends on understanding the assumptions invoked for each equation. For instance, the derivation of Equations (1) and (2) depends on identifying and exploiting the time at which a common velocity is reached on the contact surface between the vehicles. Since this common velocity is achieved prior to the restitution phase of the collision, at the time of maximum energy absorption, the appropriate crush energy value for Equations (1) and (2) is the *absorbed* crush energy. On the other hand, in Equation (38), the crush energy quantifies the difference between the pre-impact and the post-impact kinetic energies of the vehicles. This difference is, by definition, the *dissipated* energy, and thus, the dissipated energy should be used with Equation (38). Both the absorbed and the dissipated energies can form the basis of a consistent methodology, since they are related through the coefficient of restitution. This paper has shown that Equations (1) and (2) can be rewritten in a form that calls for the dissipated energy. Alternatively, Equation (38) can be written in terms of the absorbed energy.

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