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ABSTRACT

This paper describes, demonstrates and validates a method for incorporating the effects of restitution into crush analysis. The paper first defines the impact coefficient of restitution in a manner consistent with the assumptions of crush analysis. Second, modified equations of crush analysis are presented that incorporate this coefficient of restitution. Next, the paper develops equations that model restitution response on a vehicle-specific basis. These equations utilize physically meaningful empirical constants and thus improve on restitution modeling equations already in the literature of accident reconstruction. Finally, the paper presents analysis of four vehicle-to-vehicle crash tests, demonstrating that the application of the restitution model derived in this paper results in crush analysis yielding more accurate ΔV calculations.

INTRODUCTION

During the approach phase of an impact, the vehicle structure absorbs energy as it crushes to a maximum depth. This phase is followed by the restitution phase, during which the structure rebounds partially and restores some of the energy that it absorbed. This structural rebound and the associated partial restoration of energy has the effect of increasing the change in velocity (ΔV) experienced by a vehicle, over and above that which it experienced during the approach phase. The equations of crush analysis have traditionally neglected the ΔV that occurs during the restitution phase, and thus, underestimate the velocity change that a vehicle experiences during an impact.

Crush analysis theory has traditionally contained two parts. The first calculates the magnitude of crush energy absorbed by the vehicle structure. The second relates the absorbed crush energy for the two-vehicle system to the vehicle velocity changes. This paper proposes no changes to these parts of crush analysis theory. Instead, it proposes adding additional equations to crush analysis for calculating the impact coefficient of restitution and for calculating the total vehicle velocity changes, including the effects of restitution.

DEFINING THE COEFFICIENT OF RESTITUTION

In addition to dissipating energy through crushing, an impact also dissipates energy through intervehicular sliding and snagging (friction-type energy losses). As Reference 2 describes, the impact force can be broken down into components, one acting perpendicular (normal) to the intervehicular contact surface and the other acting parallel (tangential) to that surface. The component of the impact force normal to the contact surface arises due to the crushing of the vehicle structure and the component tangential to the contact surface arises due to the intervehicular sliding and snagging. It is, therefore, normal to the contact surface that the vehicle structure absorbs and restores crush energy and it is the coefficient of restitution that describes the degree to which crush energy is restored. Equation (1) defines the coefficient of restitution normal to the intervehicular contact surface.

$$\varepsilon = -\frac{V_{S,Cn}}{V_{A,Cn}} \qquad (1)$$

In Equation (1), $V_{A,Cn}$ is the normal relative approach velocity at the point of collision force transfer and $V_{S,Cn}$ is the normal relative separation velocity at the point of collision force transfer. The negative sign in Equation (1) produces a positive value for the coefficient of restitution since the separation velocity will be in the opposite direction as the approach velocity.

Reference 17 derived the following equation for calculating the approach velocity along the collision force line of action for a general two-vehicle planar impact where friction-type energy losses are negligible:

$$V_A = \sqrt{\frac{\gamma_1 M_1 + \gamma_2 M_2}{\gamma_1 M_1 \gamma_2 M_2} \cdot 2 \cdot E_A} \qquad (2)$$

In Equation (2), M_1 and M_2 are the vehicle masses, γ_1 and γ_2 are the effective mass multipliers, and E_A is the total absorbed energy for the two-vehicle system. When

friction-type energy losses are negligible, the collision force line of action coincides with the normal direction and Equation (2) yields the normal direction approach velocity. Equation (2) also assumes that the initial rotational velocities of the vehicles are negligible, and thus, it yields the approach velocity relative to the vehicle centers of gravity and the point of collision force transfer. Thus, if friction losses are neglected, Equation (2) can be substituted into Equation (1) for $V_{A,Cn}$.

The normal separation velocity at the point of collision force transfer can be written with an analogous expression as follows:[1]

$$V_{S,Cn} = -\sqrt{\frac{\gamma_1 M_1 + \gamma_2 M_2}{\gamma_1 M_1 \gamma_2 M_2} \cdot 2 \cdot E_R} \qquad (3)$$

In Equation (3), $V_{S,Cn}$ is the separation velocity along the line of action and at the point of application of the collision force. E_R is the total restored energy for the two-vehicle system. Substitution of Equations (2) and (3) into Equation (1) yields Equation (4), which is the coefficient of restitution written in terms of the absorbed and restored energies.

$$\varepsilon = \left.\sqrt{\frac{E_R}{E_A}}\right|_{\substack{negligible \\ friction \\ losses}} \qquad (4)$$

Equation (4) is valid for a two-vehicle planar impact as long as the dominant mechanism of energy loss during the impact is plastic deformation of the vehicle structure. While Equation (4) is generally considered valid only for central collisions, the effective mass concept that is used within crush analysis and to derive Equations (2) and (3) renders Equation (4) a valid definition of the coefficient of restitution for non-central impacts, as well [17].[2]

CALCULATING THE TOTAL ΔV

Once the coefficient of restitution has been calculated for an impact, then the total ΔV for each vehicle can be calculated in one of two ways. First, by multiplying the velocity change during the approach phase, ΔV_A, by one plus the coefficient of restitution, the total change in

velocity is obtained. This approach is given by Equation (5).

$$\Delta V = (1 + \varepsilon) \cdot \Delta V_A \qquad (5)$$

In Equation (5), ΔV_A is the velocity change traditionally calculated by crush analysis and, thus, using the well-known equation relating the absorbed energy to the velocity change, we can rewrite Equation (5) as follows:

$$\Delta V_1 = (1 + \varepsilon) \cdot \frac{1}{M_1} \sqrt{\frac{\gamma_1 M_1 \gamma_2 M_2}{\gamma_1 M_1 + \gamma_2 M_2} \cdot 2 \cdot E_A} \qquad (6)$$

Alternatively, the total ΔV can be obtained by calculating the approach ΔV with the absorbed energy and the separation ΔV with the restored energy and then adding these two values. This process can be written mathematically as follows:

$$\Delta V_1 = \frac{1}{M_1} \sqrt{2 \cdot \frac{\gamma_1 M_1 \gamma_2 M_2}{\gamma_1 M_1 + \gamma_2 M_2}} \cdot \left[\sqrt{E_A} + \sqrt{E_R}\right] \qquad (7)$$

Substitution of Equation (4) into Equation (7) will show that Equations (6) and (7) are equivalent. From Equation (6) it can be seen that a coefficient of restitution of 0.1 will make a 10% difference in the total ΔV. This is true even though, as Equation (4) shows, the restored energy in this case would be only 1% of the absorbed energy. This is why Equation (7) must be written with the absorbed and restored energies under separate square root signs. If they were written under the same square root sign and simply added, then the difference in ΔV from including the restored energy would only be 0.5% instead of the physically accurate 10%.[3]

VEHICLE SPECIFIC RESTITUTION RESPONSE

During an impact between two vehicles, each vehicle exhibits a unique structural response. Thus, application of identical collision forces to the vehicles results in each vehicle absorbing and restoring different quantities of energy. This being the case, Equation (4) can be used to identify three coefficients of restitution for a two-vehicle collision – a vehicle-specific coefficient of restitution for each vehicle and an overall coefficient of restitution for the impact. The vehicle-specific coefficients of restitution are analogous to the equivalent barrier speeds associated with each vehicle in the sense that they are representative values based on the structural properties and the magnitude of energy each vehicle absorbs. In fact, these coefficients of restitution will be shown to be theoretically related to the equivalent barrier speeds. In the discussion that follows, we describe the procedure

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[1] Equation (3) yields the separation speed at the point of collision force transfer, not relative to the vehicle centers of gravity. Equation (2) yields the approach velocity at the vehicle centers of gravity and at the point of collision force transfer because the initial rotational velocities of the vehicles are considered negligible. After separation, the rotational velocities may not be negligible, and the separation velocity relative to the centers of gravity will be different than at the point of collision force transfer.

[2] The effective mass multipliers used in Equations (2) and (3) are implicitly functions of all of the collision variables including the vehicle masses and inertias, the initial velocities and orientations, and the final velocities and orientations.

[3] This is contrary to statements in Reference 22.

for calculating the vehicle-specific coefficients of restitution and for combining those to obtain an overall coefficient of restitution for the collision.

Reference 5 idealized the dynamic force-crush curve for a vehicle structure as shown in Figure 1.[4] This force-crush curve has four key points that define the structural response of the vehicle. The curve begins at Point 1, where there is no crush, and therefore, no force. At Point 2, the dynamic crush, C_m, has reached its maximum value at a force level that depends on the stiffness K_1, the slope of the curve between Points 1 and 2. K_1 is assumed to be a constant. Between Points 2 and 3, the curve drops to a lower force level without any significant change in the deformation depth. Then, from Point 3 to Point 4 the structure rebounds partially and the collision force goes to zero at the residual crush value C_R.

The energy absorbed by the vehicle structure during the approach phase of the collision is equal to the area under the force-crush curve between Points 1 and 2. Similarly, the energy restored during the restitution phase is equal to the area under the force-crush curve from Point 3 to Point 4. Therefore, the absorbed and restored energies can be written as follows:

$$E_A = \frac{K_1 w_0 C_m^2}{2} \qquad (8)$$

$$E_R = \frac{K_2 w_0 (C_m - C_R)^2}{2} \qquad (9)$$

In Equations (8) and (9), w_0 is the vehicle damage width. Substitution of those equations into Equation (4) yields the following equation, which relates the coefficient of restitution to the depth of the dynamic and residual crush and the loading and unloading slopes of the force-crush curve:

$$\varepsilon = \sqrt{\frac{K_2}{K_1}} \cdot \left(1 - \frac{C_R}{C_m}\right) \qquad (10)$$

For a low severity collision where there is complete structural restitution, C_R is equal to zero and the maximum coefficient of restitution can, therefore, be written as follows:

$$\varepsilon_{max} = \sqrt{\frac{K_2}{K_1}} \qquad (11)$$

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[4] Figures appear at the end of the paper, following the References section.

Now, assume a linear relationship between the dynamic and residual of the following form [9, 10]:

$$C_R = \zeta C_m - \frac{A}{B} \qquad (12)$$

In Equation (12), ζ is a constant that defines the slope of the relationship between residual and dynamic crush. A and B are the stiffness coefficients utilized in crush analysis. Substitution of Equations (11) and (12) into (10) yields the following expression for the coefficient of restitution:

$$\varepsilon = \varepsilon_{max}\left(1 - \zeta \frac{BC_R}{BC_R + A}\right) \qquad (13)$$

Equation (13) relates the vehicle-specific coefficient of restitution to the depth of residual crush and the structural properties (A, B, ε_{max}, and ζ). The next section explores the physical significance of ε_{max} by writing Equation (13) in a form that depends on the equivalent barrier speed (EBS) instead of the residual crush.

RESTITUTION AND EBS

We begin by rewriting Equation (12) as follows:[5]

$$C_m - C_R = (1 - \zeta) \cdot C_m + \frac{A}{B} \qquad (14)$$

Substitution of Equation (14) into (9), followed by substitution of the resulting equation and Equation (8) into Equation (4), yields Equation (15).

$$\varepsilon = \varepsilon_{max}\left(1 - \zeta + \frac{A}{BC_m}\right) \qquad (15)$$

Now, observe that for a barrier impact, Equation (2) can be written as follows:

$$V_A = EBS = \sqrt{\frac{2 \cdot E_A}{M}} \qquad (16)$$

Substitution of Equation (8) into Equation (16) yields the following expression for the dynamic crush:

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[5] Equation (14) yields an additional interpretation of ζ. Since increasing crash severity leads to increasing dynamic crush, the value of ζ describes the extent to which the difference between residual and dynamic crush is dependent on impact severity. The closer ζ is to a value of 1, the less dependent the difference between residual and dynamic crush is on crash severity.

$$C_m = EBS\sqrt{\frac{M}{K_1 w_0}} \qquad (17)$$

Figure 2 shows the residual force-crush model of CRASH side-by-side with the first leg of the dynamic force model of Figure 1. These two models should yield equivalent estimates of the absorbed crush energy [19]. Thus, the absorbed crush energy obtained from the equations of the CRASH model can be equated with Equation (8), the absorbed crush energy obtained from the dynamic force model. Equation (18) equates these two absorbed energy values on a per unit width basis.

$$\frac{B}{2}C_R^2 + AC_R + \frac{A^2}{2B} = \frac{K_1 C_m^2}{2} \qquad (18)$$

Substitution of Equation (13) into (18) to eliminate C_m yields the following expression:

$$\left(B\zeta^2 - K_1\right)\cdot\left(\frac{B}{2}C_R^2 + AC_R + \frac{A^2}{2B}\right) = 0 \qquad (19)$$

In Equation (19), the second term is equal to the absorbed energy per unit width from the residual crush model of CRASH and, thus, this term will only be equal to zero in the trivial case when there is no residual crush. Satisfaction of Equation (19), therefore, requires that the first term in parenthesis be equal to zero, yielding the following relationship:

$$\zeta = \sqrt{\frac{K_1}{B}} \qquad (20)$$

Equation (20) provides a relationship between the dynamic crush stiffness coefficient K_1 and the residual crush stiffness coefficient B. Substitution of Equation (20) into (17) yields an expression for the dynamic crush, which can then be substituted into Equation (15) to yield the following equation:

$$\varepsilon = \varepsilon_{max}\left(1 - \zeta\left\{1 - \frac{1}{EBS}\sqrt{\frac{A^2}{B}\frac{w_0}{M}}\right\}\right) \qquad (21)$$

From the equations in Reference 12, it can be shown that:

$$\frac{A^2}{B} = \frac{M b_0^2}{w_0} \qquad (22)$$

Substitution of Equation (22) into (21) yields the following equation relating the coefficient of restitution to the EBS:

$$\varepsilon = \varepsilon_{max}\left[1 - \zeta\left(1 - \frac{b_0}{EBS}\right)\right] \qquad (23)$$

Equation (23) reveals that the value of ε_{max} is realized when the EBS is equal to the damage offset speed, b_0. Thus, the value of ε_{max} physically corresponds to the vehicle-specific restitution value from a barrier impact at a speed of b_0, which is the EBS at the onset of residual crush. The value of b_0 usually falls within a range of 4 to 5 mph for frontal impacts [12, 20].

CALCULATING VEHICLE-SPECIFIC ε_{max} AND ζ VALUES

Reference 9 derives the following equation to describe the relationship between the dynamic crush and the coefficient of restitution:

$$\varepsilon = \frac{\Gamma}{C_m} + \rho \qquad (24)$$

Using the equations presented in References 9 and 10, it can be shown that the empirical constants Γ and ρ of Equation (24) are related to the empirical constants ε_{max} and ζ through the following equations:

$$\rho = \varepsilon_{max}\left(1 - \zeta\right) \qquad (25)$$

$$\Gamma = \varepsilon_{max}\frac{A}{B} \qquad (26)$$

Equations (25) and (26) fail to reveal a clear physical meaning for the empirical constants Γ and ρ. Thus, the restitution model derived in this paper improves on that derived in References 9 and 10 by using the physically meaningful constants, ε_{max} and ζ.

Reference 10 describes a method for determining Γ and ρ from crash test data. This method can be modified slightly to yield values for ε_{max} and ζ. The analyst first calculates the following values from a crash test: (1) C_{m1}, the maximum dynamic crush of the vehicle structure; (2) $(C_R/C_m)_1$, the ratio of residual crush to maximum dynamic crush; (3) ε_1, the coefficient of restitution; and (4) the A and B stiffness coefficients. Once these four items are known, Equations (27) and (28) can be used to obtain ε_{max} and ζ.

$$\varepsilon_{max} = \frac{\varepsilon_1}{1 - \left(\dfrac{C_R}{C_m}\right)_1} \qquad (27)$$

$$\zeta = \frac{1}{C_{m1}}\left(C_{R1} + \frac{A}{B}\right) \qquad (28)$$

Reference 14 raises the concern that Equation (27) may yield a value of ε_{max} greater than 1.0, a condition that is inappropriate from a physical standpoint. This situation occurs when the following condition is met:

$$1 - \varepsilon_1 < \left(\frac{C_R}{C_m}\right)_1 \qquad (29)$$

Some frontal barrier impact crash tests do yield ε_{max} values greater than 1.0. In our experience, this occurs in cases when the dynamic force-crush curve of Figure 1 does not adequately model the actual force-crush curve.

To demonstrate the use of Equations (27) and (28) to calculate vehicle-specific ε_{max} and ζ values, we will now examines restitution data from seven full-overlap frontal barrier impact crash tests of Chevrolet Astro vans (MY 1985-96).[6] For each test, acceleration data was downloaded from the NHTSA crash test database for the longitudinal accelerometers that were rearward of the crushing region of the vehicle.[7] The acceleration signals downloaded for each test were filtered in accordance with the recommendations of SAE J211. Figure 3 shows the resulting acceleration versus time curve for the first test.[8]

After filtering, the acceleration signals were integrated to obtain velocity curves and, from the velocity data, the coefficient of restitution for each test was calculated. Figure 4 shows the velocity versus time curve for the first test. The coefficient of restitution for each test was calculated using the initial velocity and the maximum rebound velocity [11]. For the test shown in Figure 4, the initial velocity was 34.8 mph (56.0 kph) and the maximum rebound velocity was 3.9 mph (6.3 kph), yielding a coefficient of restitution of 0.111.

Finally, the acceleration signals were integrated again to obtain the displacement and this data was used to plot the force-crush curve for each test. The force per unit width resulted from multiplying the filtered acceleration

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[6] NHTSA Test Numbers: 800, 1677, 1692, 1979, 2046, 2071, 2404.

[7] http://www-nrd.nhtsa.dot.gov/database/nrd-11/veh_db.html

[8] NHTSA Test Number 800

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signals by the vehicle weight at each time step and then dividing by the vehicle width. Figure 5 shows the force-crush curve for the first test. In this case, the maximum dynamic crush was 24.5 inches (622 mm). Neptune Engineering reported an air-gap adjusted average residual crush measurement of 20.3 inches (516 mm) for this test, yielding a ratio of residual to maximum dynamic crush of approximately 0.83. Following this analysis of the accelerometer data, Equations (28) and (29) were applied to obtain values of ε_{max} and ζ. In the case of the first test, this yielded an ε_{max} value of 0.65 and a ζ value of 0.97.

Table 1 lists the initial speed, coefficient of restitution, ratio of residual to maximum dynamic crush, and calculated test-specific ε_{max} and ζ values for each of the seven Astro Van crash tests.[9] The last row of the table lists average values for ε_{max} and ζ. For calculating the test-specific ζ values listed in Table 1, the authors used Neptune Engineering's air-gap adjusted residual crush measurements and A and B stiffness coefficients for this vehicle.[10] The ε_{max} and ζ values listed in Table 1 have average values of 0.75 and 0.94, respectively. Figure 6 shows a plot of Equation (13) using these average values. The restitution values listed in Table 1 have also been plotted, with unfilled circles.

COMBINING VEHICLE SPECIFIC RESTITUTION COEFFICIENTS

Now, consider several approaches for combining vehicle-specific coefficients of restitution to obtain the overall impact coefficient of restitution. References 4 and 9 recommend the first approach, given by Equation (30), which we will call the *energy equivalent* approach.

$$\varepsilon_{energy} = \sqrt{\varepsilon_1^2 \frac{E_{A1}}{E_A} + \varepsilon_2^2 \frac{E_{A2}}{E_A}} \qquad (30)$$

Brach has suggested two additional approaches [2]. The first of these he termed the *stiffness equivalent* approach, suggesting the following equation:

$$\varepsilon_{stiffness} = \sqrt{\frac{\varepsilon_1^2 K_{1,2} + \varepsilon_2^2 K_{1,1}}{K_{1,1} + K_{1,2}}} \qquad (31)$$

In Equation (31), $K_{1,1}$ is the dynamic loading stiffness for Vehicle #1 and $K_{1,2}$ is the dynamic loading stiffness for Vehicle #2 [See Figure 2]. The second approach

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[9] Tables are shown at the end of the paper following the Figures section.

[10] www.neptuneeng.com, Vehicle Crush Stiffness Coefficients Reference Number: VanAst01A.

suggested by Brach he termed the *mass equivalent* approach and he derived the following equation:

$$\varepsilon_{mass} = \sqrt{\frac{\varepsilon_1^2 M_2 + \varepsilon_2^2 M_1}{M_1 + M_2}} \qquad (32)$$

In the next section, Equations (30) through (32) will be examined within the context of vehicle-to-vehicle crash test data to determine which yields the most accurate impact coefficient of restitution.

VALIDATION STUDY

This section presents analysis of vehicle-to-vehicle crash test data and demonstrates that considering restitution in crush analysis results in improved accuracy over crush analysis that neglects restitution. Equations (6), (23), (30), (31) and (32) are applied, in conjunction with other well-known equations of crush analysis, to analyze staged collisions for which the vehicle impact speeds are known and for which the ΔVs and impact coefficient of restitution can be obtained. Comparisons are made between the actual and calculated ΔVs and restitution coefficients. The analysis presented in this section utilized the following procedure:

- Data from four vehicle-to-vehicle crash tests was obtained from NHTSA's research and development website.[11] This data included the test report, pre and post-test photographs, video footage, and accelerometer data. The four crash tests chosen for this study were offset, angled impacts between SUVs and Honda Accords.

- To obtain the center of gravity ΔV, the post-impact yaw velocity, and the post-impact travel direction for each vehicle, the accelerometer data was analyzed in accordance with the equations derived in References 3, 7 and 21. Prior to analysis, the accelerometer data was filtered in accordance with SAE J211 using software that is publicly available on NHTSA's research and development website.[12] Integration of the accelerometer data utilized Simpson's Rule.

- The equations of References 3, 7, and 21 required that, for each vehicle, two accelerometer locations be specified. Our analysis of the accelerometer data for each test utilized data from accelerometers mounted on the vehicle sills. Three of the four test reports did not specify the exact locations of the accelerometers, but they did give the distances between the front and rear sill accelerometers. Based on the distance

[11] http://www-nrd.nhtsa.dot.gov/database/nrd-11/veh_db.html.

[12] http://www-nrd.nhtsa.dot.gov/software/Index.htm

between accelerometers and on the vehicle geometry, accelerometer locations were estimated.

- For each vehicle, there were multiple accelerometer combinations that could be used for the analysis. In each case, multiple accelerometer combinations were analyzed and the resultant ΔV results from these accelerometer combinations were averaged. Analysis of video from the crash tests was used in conjunction with the accelerometer signals to determine an accurate post-impact yaw rate for the vehicles.

- Based on data obtained from the crash test report, photographs and video footage, the equations of planar impact mechanics [2] were applied to determine the coefficient of restitution for each test. Within the planar impact mechanics analysis, the impact center location and coefficient of restitution were optimized to match the ΔVs, post-impact rotation rates and post-impact travel directions determined from the accelerometer data. Once the planar impact solution was optimized to match these known values, the coefficient of restitution from the planar impact mechanics analysis was considered the coefficient of restitution for the impact.

- Based on data from the crash test report, pictures and video, the equations of crush analysis were applied in conjunction with the restitution model derived in this paper. In each case, crush measurements taken at the bumper beam, without the bumper fascia, were utilized for the crush analysis. The collision force directions (PDOF) and moment arms determined during the planar impact mechanics analysis were used for this crush analysis.

- Crush analysis yielded calculated values for the vehicle ΔVs and the impact coefficient of restitution. These crush analysis calculations included calculating vehicle-specific coefficients of restitution for each vehicle using Equation (23) and combining of those vehicle-specific coefficients using Equations (30) through (32). For each test, the ΔVs calculated from crush analysis were compared to the actual ΔVs obtained from the accelerometer data. Finally, the coefficients of restitution predicted by Equations (31), (32) and (33) were compared to the actual coefficient of restitution obtained with the equations of planar impact mechanics.

NHTSA TEST #4363

First, consider NHTSA Test #4363, a collision between a 1997 Chevrolet Blazer and a 1997 Honda Accord. The vehicles in the test were aligned such that front of the Blazer would impact the front left corner of the Accord at an angle of approximately 30 degrees (Figure 7). The Blazer weighed 4,695 pounds (2130 kg) and, at impact,

was traveling 35.0 mph (56.3 kph). The Accord weighed 3,451 pounds (1565 kg) and, at impact, was traveling 35.1 mph (56.5 kph).

Figure 8 is a series of images from the test video that depict the collision dynamics for this test. The Accord, initially traveling from left to right in the frames, experienced a change in velocity direction of approximately 90 degrees and at the time of separation was traveling almost directly downward in the frames. The impact generated little overall rotation of the Honda. The Blazer, on the other hand, experienced a relatively significant change in rotational velocity during the impact.

Figures 9 through 12 depict the results of analyzing the accelerometer data for this test. Figure 9 shows the resultant ΔV for the Blazer throughout the impact. The plot of Figure 9 was constructed by averaging the resultant ΔV plots from the following three accelerometer combinations: left rear sill/right front sill, left rear sill/right rear sill, and right front sill/right rear sill. As the figure shows, the accelerometer data yielded a maximum resultant velocity change for the Blazer of approximately 29.1 mph (46.8 kph). Figure 10 is a similar plot for the Accord. This plot was constructed by averaging the resultant ΔV plots from the following three accelerometer combinations: left rear sill/right front sill, left rear sill/right rear sill, and right front sill/right rear sill.

Figures 11 and 12 depict the yaw velocity for the two vehicles throughout the impact. The plots were constructed using by averaging the yaw velocities obtained from the same accelerometer combinations used for the ΔV analysis. As these plots show, the yaw velocity of the Blazer at the time of the maximum resultant ΔVs was approximately 197 degrees per second. At this same time, the Accord had a yaw velocity of approximately 48 degrees per second.[13]

Figure 13 depicts the planar impact mechanics calculations performed for this test. Within the spreadsheet depicted in Figure 13, dark gray cells represent user inputs and light gray cells represent calculated values. As the figure shows, the inputs for planar impact mechanics include the vehicle weights, yaw moments of inertia, the impact center location relative to the vehicle centers of gravity, the vehicle heading angles, the initial vehicle velocities, the orientation of the collision normal direction (Γ), the impulse ratio as a percentage of the critical impulse ratio, and the coefficient of restitution. Refer to Reference 2 for the equations of planar impact

mechanics and for a detailed description of the role that each of these parameters plays in those equations.

In this case, a coefficient of restitution of 0.126, along with the impact center location and contact plane orientation shown in Figure 14, resulted in an excellent match with the ΔVs and post-impact rotation rates from the accelerometer analysis. In addition to yielding the coefficient of restitution, the planar impact mechanics solution can also be used to obtain the approximate approach phase ΔVs for the Blazer and the Accord. Setting the coefficient of restitution in the optimized planar impact mechanics solution to 0.000 yielded approach phase velocity changes for the Blazer and the Accord of 26.6 mph (42.8 kph) and 36.1 mph (58.1 kph), respectively.

Figures 15 through 17 depict crush analysis calculations for this test. For all four of the tests analyzed in this paper, crush analysis calculations were conducted in a Microsoft Excel workbook, which contained an input worksheet for each vehicle and an output worksheet. Figure 15 and 16 depict input worksheets for one of the crush analysis scenarios conducted for this particular case. Again, those cells that call for a user input are colored dark gray while cells containing calculated values are colored light gray. As the images of Figures 15 and 16 show, in addition to the crush measurements and vehicle weight, each input sheet calls for the vehicle yaw moment of inertia, the collision force moment arm, the collision force direction, the deformation location, ε_{max} and ζ, b_0 and the A and B stiffness coefficients.

Calculations were conducted to estimate the yaw moment of inertia for each vehicle [1, 6] and the collision force direction and moment arm for each vehicle were obtained from the planar impact mechanics solution. Air-gap adjusted crush stiffness coefficients for each vehicle were purchased from Neptune Engineering and restitution parameters, ε_{max} and ζ, were calculated for each vehicle. In this case, the ε_{max} and ζ values for the Blazer were 0.62 and 0.87, respectively, and they were 0.66 and 0.85 for the Accord.

Each vehicle input sheet also contains intermediate calculations, which include the vehicle mass, the radius of gyration, the effective mass multiplier, the adjusted mass, the oblique collision energy correction factor,[14] the zone-specific collision force and absorbed energy, the total damage width, the total force and absorbed energy, the ratio of the Vehicle 1 collision force to the Vehicle 2 collision force, the equivalent barrier speed,

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[13] The coordinate system used in the NHTSA tests is a left-handed coordinate system. The coordinate system used in the planar impact mechanics analysis described below is a right-handed coordinate system. Thus, the sign of the yaw rates shown in Figures 11 and 12 have the opposite sign as what they have in the planar impact mechanics calculations.

[14] The crush analysis calculations used in this paper utilized the (1/cos α) energy correction factor. See Reference 18 for a discussion of four oblique impact correction factor alternatives. It is important to note that this correction factor is different that that utilized in CRASH3 and EDCRASH, and therefore, those programs would yield results different from those reported below.

and the vehicle-specific coefficient of restitution. The vehicle-specific coefficients of restitution are calculated using Equation (23). The force ratio is included on the worksheets so that satisfaction of Newton's third law could be verified during the analysis [13].

For this case, two sets of A and B stiffness coefficients were available for each of the vehicles – one set derived from full-overlap impact tests and one derived from offset impact tests. Using these four sets of stiffness coefficients, three sets of crush analysis calculations were generated for this test. These three crush analysis scenarios had the following combinations of stiffness coefficients:

- Blazer Offset Impact Stiffness Coefficients combined with Full-Overlap Stiffness Coefficients for the Accord.

- Blazer Full-Overlap Stiffness Coefficients with the Accord Stiffness Coefficients set to achieve force balance.

- Accord Offset Impact Stiffness Coefficients with the Blazer Stiffness Coefficients set to achieve force balance.

With the exception of the stiffness coefficients, all crush analysis inputs were held fixed between these three scenarios.

The crush analysis scenario that utilized offset impact stiffness coefficients for the Accord yielded the most accurate prediction of the actual approach phase ΔVs. Figure 17 depicts the crush analysis outputs for this scenario. As the figure shows, this scenario predicted an approach phase ΔV of 26.1 mph (42.0 kph) for the Blazer and 35.5 mph (57.1 kph) for the Accord. These calculated values are approximately 2 percent below the actual approach phase velocity changes.

Figure 17 also shows the outputs for the coefficients of restitution obtained from Equations (30) through (32), based on the vehicle specific coefficients of restitution calculated within the sheets shown in Figures 15 and 16. In this case, all three equations yielded a coefficient of restitution of approximately 0.164, 30 percent higher than the actual value of 0.126. Using this value for the coefficient of restitution yielded total ΔVs 30.4 mph (48.9 kph) and 41.3 mph (66.5 kph) for the Blazer and the Accord, respectively. These values are 4.6 percent higher the actual ΔVs. Had restitution been neglected in these crush analysis calculations, the ΔVs would have been approximately 10 percent low.

Utilizing offset impact stiffness coefficients for the Blazer[15] and full-overlap stiffness coefficients for the Accord,[16] along with the crush measurements and damage width as they were reported in the crash test report resulted in calculated impact forces that were within 1 percent of balancing. This second crush analysis scenario resulted in approach phase ΔVs of 24.2 mph (38.9 kph) and 32.7 mph (52.6 kph) for the Blazer and the Accord, respectively. These values are approximately 9.4 percent lower than the actual approach phase ΔVs. In this case, Equations (30) through (32) each yielded a value of 0.170 for the overall coefficient of restitution, a value that is 34.9 percent higher than the actual value. Finally, this scenario resulted in total ΔVs of 28.3 mph (45.5 kph) and 38.5 mph (62.0 kph) for the Blazer and the Accord, respectively. These values are within 3 percent of the actual values. So, in this case, the crush analysis calculations underestimated the approach phase ΔVs, overestimated the restitution coefficient, and these results combined to produce a relatively accurate prediction of the total ΔV. Had restitution been neglected in these crush analysis calculations, the calculated ΔVs would have been 17 percent low.

The final crush analysis scenario conducted for this test, which utilized full-overlap stiffness coefficients for the Blazer, resulted in the least accurate calculated ΔVs, overestimating the actual ΔVs by approximately 17.5 percent. Considering the impact configuration and the damage to the vehicles, it could be anticipated that using offset impact stiffness coefficients would yield better accuracy than full-overlap impact stiffness coefficients [15]. Not only is the impact an offset impact for the Accord, but the Blazer also experienced damage consistent with an offset impact. Figures 18 and 19, which are post-test photographs of the undercarriages of the Blazer and the Accord, respectively, show this damage to the Blazer and the Accord.

NHTSA TEST #4364

Now, consider NHTSA Test #4364, an impact between a 2002 Chevrolet Trailblazer and a 1997 Honda Accord. The impact configuration for this test was identical to that of all four of the tests considered in this paper, with the front of the Trailblazer aligned to impact the front left corner of the Accord. Figure 20 is a series of images from the test video that depict the collision dynamics. In this case, analysis of the accelerometer data showed that the Trailblazer experienced a ΔV of 26.6 mph (42.8 kph) and had a resulting post-impact rotation rate of 136

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[15] www.neptuneeng.com, Vehicle Crush Stiffness Coefficients Reference Number: Blazer13X.

[16] www.neptuneeng.com, Vehicle Crush Stiffness Coefficients Reference Number: Accord33A.

degrees per second. The accelerometer data for the Accord yielded questionable results when integrated and so the requirement of momentum conservation was used to obtain a ΔV of 40.1 mph (64.5 kph) for the Accord. Planar impact mechanics calculations for this case yielded an impact coefficient of restitution of 0.089.

In this case, there were three sets of stiffness coefficients available – full-overlap stiffness coefficients for the Trailblazer and both full-overlap and offset stiffness coefficients for the Accord. Three crush analysis scenarios were generated with these stiffness coefficients. The first scenario utilized full overlap stiffness coefficients for the Accord,[17] the second scenario utilized offset impact stiffness coefficients for the Accord,[18] and the third utilized full-overlap stiffness coefficients for the Trailblazer. For each scenario, the stiffness coefficients for the remaining vehicle were adjusted to achieve force balance. For all three scenarios, the ε_{max} and ζ values for the Trailblazer were 0.49 and 0.90, respectively, and they were 0.66 and 0.85 for the Accord.

In this case, the crush analysis scenario that utilized full-overlap stiffness coefficients for the Trailblazer yielded the most accurate calculation of the approach phase ΔVs. The calculated approach phase ΔVs for this scenario were 24.1 mph (38.8 kph) and 36.2 mph (58.3 kph) for the Trailblazer and Accord, respectively – values that are 2.9 percent below the actual approach phase velocity changes. For this crush analysis scenario, Equations (30) through (32) yielded nearly identical values for the impact coefficient of restitution. The average calculated coefficient of restitution for this crush analysis scenario was 0.146, a value that was 64.0 percent higher than the actual value. Therefore, the calculated total ΔVs for this scenario were 27.6 mph (44.4 kph) and 41.5 mph (66.8 kph) for the Trailblazer and the Accord, respectively – values that were 3.6 percent high. Had restitution been neglected in this instance, the calculated ΔVs would have been 9.6 percent low.

The crush analysis scenario that utilized offset stiffness coefficients for the Accord produced the best calculated total ΔVs – 26.6 mph (42.8 kph) and 40.0 mph (64.4 kph) for the Trailblazer and the Accord, respectively – 0.1 percent below the actual values. The average calculated coefficient of restitution for this scenario was approximately 0.148 and there was little variance between the coefficient of restitution calculated with each of the three approaches. Had restitution been neglected in this crush analysis scenario, the calculated ΔVs would have been low by 13.0 percent.

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[17] www.neptuneeng.com, Vehicle Crush Stiffness Coefficients Reference Number: Accord33A.

[18] www.neptuneeng.com, Vehicle Crush Stiffness Coefficients Reference Number: Accord26X.

NHTSA TEST #4474

Next, consider NHTSA Test #4474, an impact between a 1999 Mitsubishi Montero Sport and a 1997 Honda Accord, with the front of the Montero aligned to impact the front left corner of the Accord. Figure 21 is a series of images from the test video that depict the collision dynamics. Analysis of the accelerometer data showed that the Montero experienced a ΔV of 28.5 mph (45.8 kph) with a resulting post-impact rotation rate of 192 degrees per second. The accelerometer data for the Accord yielded a velocity change of 40.7 mph (65.5 kph) with little post-impact rotation. Planar impact mechanics yielded an impact coefficient of restitution of 0.150.

In this case, utilizing offset impact stiffness coefficients for the Accord again yielded the best results. The ε_{max} and ζ values for the Montero were 0.42 and 0.86, respectively, and they were 0.66 and 0.85 for the Accord. These crush analysis calculations resulted in calculated ΔVs for the Montero and the Accord of 25.8 mph (41.5 kph) and 36.3 mph (58.4 kph), respectively. Therefore, the calculated ΔVs underestimated the actual ΔVs by around 10 percent. In this instance, had restitution been neglected, crush analysis would have yielded ΔVs that were about 22 percent low. The crush analysis calculations yielded an average coefficient of restitution of 0.157, approximately 5% high. There was little variance between the values obtained from each of Equations (30) through (32).

NHTSA TEST #4438

Finally, consider NHTSA Test #4438, a collision between a 2001 Mitsubishi Montero Sport and a 1997 Honda Accord, again, with the front of the Montero aligned to impact the front left corner of the Accord. Figure 22 is a series of images from the test video that depict the collision dynamics. Analysis of the accelerometer data showed that the Montero experienced a ΔV of 27.6 mph (44.4 kph) with a resulting post-impact rotation rate of 160 degrees per second and that the Accord experienced a ΔV of 39.9 mph (64.2 kph) with a resulting post-impact rotation rate of approximately 42 degrees per second. Planar impact mechanics yielded an impact coefficient of restitution of 0.0.095.

For this case, two sets of A and B stiffness coefficients were available for each of the vehicles – one set derived from full-overlap impact tests and one derived from offset impact tests. Four crush analysis scenarios were run with these stiffness coefficients. In all four of these scenarios, the ε_{max} and ζ values for the Montero were 0.42 and 0.86, respectively, and they were 0.66 and 0.85 for the Accord.

Utilizing Accord stiffness coefficients from an offset impact test yielded the most accurate results in this

case. This crush analysis scenario resulted in calculated approach phase ΔVs of 24.7 mph (39.7 kph) and 35.3 mph (56.8 kph) for the Montero and the Accord, respectively – values that are approximately 4.3 percent low. The average calculated coefficient of restitution for this scenario was 0.148 – a value that is 55.8 percent high. Finally, this crush analysis scenario resulted in calculated total ΔVs for the Montero and the Accord of 28.4 mph (45.7 kph) and 40.5 mph (65.2 kph), respectively. These calculated ΔVs are within 3 percent of the actual ΔVs. Had restitution been neglected in this instance, crush analysis would have yielded ΔVs that were about 11 percent low.

DISCUSSION

CALCULATED ΔVS

Table 2 summarizes the results from the validation portion of this paper. For each crash test, Table 2 includes the crush analysis scenario that yielded the most accurate calculated ΔVs. The fourth column of the table includes not only the percent difference between the actual and calculated ΔVs, but also lists in parenthesis the percent difference that would have existed in the calculated ΔV had restitution been neglected. These results are encouraging considering that in each case the inclusion of restitution resulted in significantly improved prediction of the actual ΔVs. Not only that, in three of the four cases, the inclusion of restitution resulted in calculated ΔVs that were within 3% of the actual values.

The crush analysis calculations carried out for this paper utilized bumper crush measurements from the crash test reports, taken without the bumper fascia in place. For the Honda Accords in these crash tests, the bumper crush measurements included both direct and induced bumper deformation. For each scenario, we included both direct and induced bumper damage regardless of whether we were utilizing stiffness coefficients determined from full-overlap or offset impact crash tests. The issue of whether crush analysis should utilize direct damage only or direct plus induced damage seems never to have been definitively resolved in the literature. The answer ultimately seems to depend on one's procedure for deriving stiffness coefficients and on how crush measurements are gathered. Strictly speaking, since Neptune Engineering derives their stiffness coefficients using direct damage only, crush analysis utilizing these stiffness coefficients should utilized direct damage only. However, review of Figures 8, 20, 21 and 22 reveals that for the Honda Accords in these tests, the direct damage region extends along the side of the vehicle, as far back as the B-pillar. Thus, using bumper crush for the crush analysis calculations ignores a significant portion of the direct damage region. This is likely why the inclusion of induced bumper damage

resulted in accurate crush analysis calculations – the inclusion of induced bumper damage compensates for the neglect of the portion of the direct damage region that would not be captured with bumper crush alone.

In addition to using unmodified bumper crush measurements from the test reports, each crush analysis calculation carried out for this paper utilized the collision force directions and moment arms obtained from planar impact mechanics analysis. This procedure would be expected to yield better crush analysis results than one that simply utilized visual estimates of the collision force direction based on vehicle damage. Over the years, the literature has often observed that the accuracy of crush analysis calculations depends on the accuracy of the analyst's estimate of the collision force. This can be problematic and produce inaccurate results if the analyst bases their estimate only on observation of the vehicle damage. However, estimates of the collision force direction can be accurate if they consider not only the vehicle damage, but also the motion of the vehicles caused by the impact forces (i.e., the direction and magnitude of the vehicle rotations). In some cases, the analyst may have no choice but to rely on such an estimate and the analysis may not be significantly degraded by such an estimate.

CALCULATED ENERGIES

There is a difference between crush analysis and planar impact mechanics in terms of how they deal with the total energy dissipation during an impact. Planar impact mechanics calculates and considers energy dissipation due to crushing, sliding and snagging [2]. Crush analysis theory, on the other hand, ignores the friction-type energy losses that occur due to intervehicular sliding and snagging [18]. Within crush analysis, the deformation energy is calculated from crush measurements taken perpendicular to the original shape of the damaged surface and then that energy is corrected for the actual direction of the collision force. Reference 18 described and discussed four alternative multipliers for correcting the calculated deformation energy and the reader is referred there for a full discussion. In theory, two of these multipliers attempt to incorporate friction-type energy losses into crush analysis and two do not. Ideally, a multiplier for the crush energy would correct the nominal crush energy to include all of the crush energy and also the friction-type energy losses. For the analysis so far reported in this paper, we utilized the following damage energy correction multiplier:

$$\frac{1}{\cos\alpha} \tag{33}$$

In Equation (33), α is the angle between the actual collision force direction and the normal to the original shape of the damaged vehicle surface. This multiplier,

which we will refer to as the Woolley multiplier, makes no theoretical attempt to incorporate friction-type losses.

The effectiveness of using this damage energy correction multiplier can be evaluated by reviewing Table 3. This table lists the actual collision energy loss as determined from the equations of planar impact mechanics. The energy loss is then further broken down into energy loss along the normal direction (crush energy) and energy loss along the tangential direction (friction-type energy losses due to snagging and sliding). Finally, for each test, the table lists the dissipated crush energy obtained from the crush analysis scenario that resulted in the most accurate prediction of the actual ΔVs. In each case, the dissipated crush energy calculated during crush analysis overestimated the actual dissipated crush energy, but underestimated the total dissipated energy. Thus, the energy correction multiplier of Equation (33) resulted in dissipated crush energy values that partially compensate for the fact that crush analysis theory does not incorporate friction-type energy losses – despite the fact that the derivation of this multiplier makes no attempt to incorporate these additional losses [18]. The overcorrection of the crush energy accomplished by Equation (33) is not, however, sufficient to fully compensate for the neglect of friction type energy losses.

For the four cases considered here, we could improve the degree to which crush analysis compensates for its neglect of friction-losses by using the CRASH 3 energy correction multiplier, given by Equation (34).

$$\frac{1}{\cos^2 \alpha} \qquad (34)$$

Table 3 includes dissipated energy values calculated with the CRASH 3 correction factor. This multiplier does result in improved predictions of the total dissipated energy for each collision. The CRASH 3 multiplier has been criticized in the literature for being excessive. In these four cases, however, these criticisms are not born out.

CALCULATED COEFFICIENTS OF RESTITUTION

For all four cases presented in this paper, the crush analysis calculations resulted in calculated coefficients of restitution that were too high. The only calculated coefficient of restitution that was close to the actual value was that from Test #4474, the test for which the calculated ΔVs exhibited the worst accuracy. There are at least two reasons that caused the restitution coefficients to be overestimated. First, the crush analysis calculations underestimated the total energy loss during the impact, as discussed in the previous paragraphs. Given that the coefficient of restitution decreases with increasing energy absorption, underestimating the

energy loss leads directly to overestimating the coefficient of restitution. On the other hand, as Table 3 also shows, when we improve the damage energy calculations by using a different multiplier, the calculations of the coefficients of restitution do not improve significantly.

This seems to indicate either that the restitution curves used for these crush analysis scenarios were not adequately sensitive to the damage energy, in the region of interest, or that the values of ε_{max} and ζ that we used were not representative of the structures and the impact mode involved in these four crash tests. All of the most accurate crush analysis scenarios utilized offset impact stiffness coefficients while using ε_{max} and ζ values obtained from full-overlap frontal barrier collisions. Further research is necessary to develop a technique to calculate ε_{max} and ζ from offset impact tests. It stands to reason that the value of ε_{max} and ζ would depend on the impact mode and it may be that using offset impact tests would have resulted in lower calculated coefficients of restitution.

As far as Equations (30) through (32) go, in all four cases presented here, there was little difference between the values calculated from these three equations. Additional case studies would be necessary to determine whether this will always be the case, but we can tentatively conclude that these three equations can be used interchangeably.

FURTHER RESEARCH

This study is clearly limited in that it considered only four staged collisions, all with an identical impact configuration and an identical make and model target vehicle. The restitution model developed in this paper produced results encouraging enough to warrant further research and development through consideration of additional case studies. The procedure that we used to obtain actual impact coefficients of restitution also deserves further research and development. In this paper, comparison has been made between coefficients of restitution calculated from crush analysis calculations and those determined by obtaining a best-fit planar impact mechanics solution for each crash test. The coefficients of restitution obtained from the best-fit planar impact mechanics analysis have been treated as the actual value. However, during the course of this research, it became clear that there can be considerable uncertainty associated with obtaining a best-fit planar impact mechanics solution for a vehicle-to-vehicle crash test. The uncertainties inherent in determining the "actual" coefficient of restitution for a vehicle-to-vehicle crash test deserve further exploration.

CONCLUSIONS

- When used in conjunction with the Woolley energy correction multiplier, the restitution model presented in this paper resulted in significantly improved ΔV calculations for the four crash tests considered in this paper.

- The CRASH 3 energy correction multiplier resulted in better calculations of the total dissipated energy for these collisions than did the Woolley multiplier.

- The restitution model presented in this paper tended to overestimate the actual coefficient of restitution.

- While it improved the prediction of the dissipated energy, the CRASH 3 multiplier did not significantly improve the calculated coefficients of restitution.

- Further research is necessary to determine whether these conclusions can be generalized.

ACKNOWLEDGMENTS

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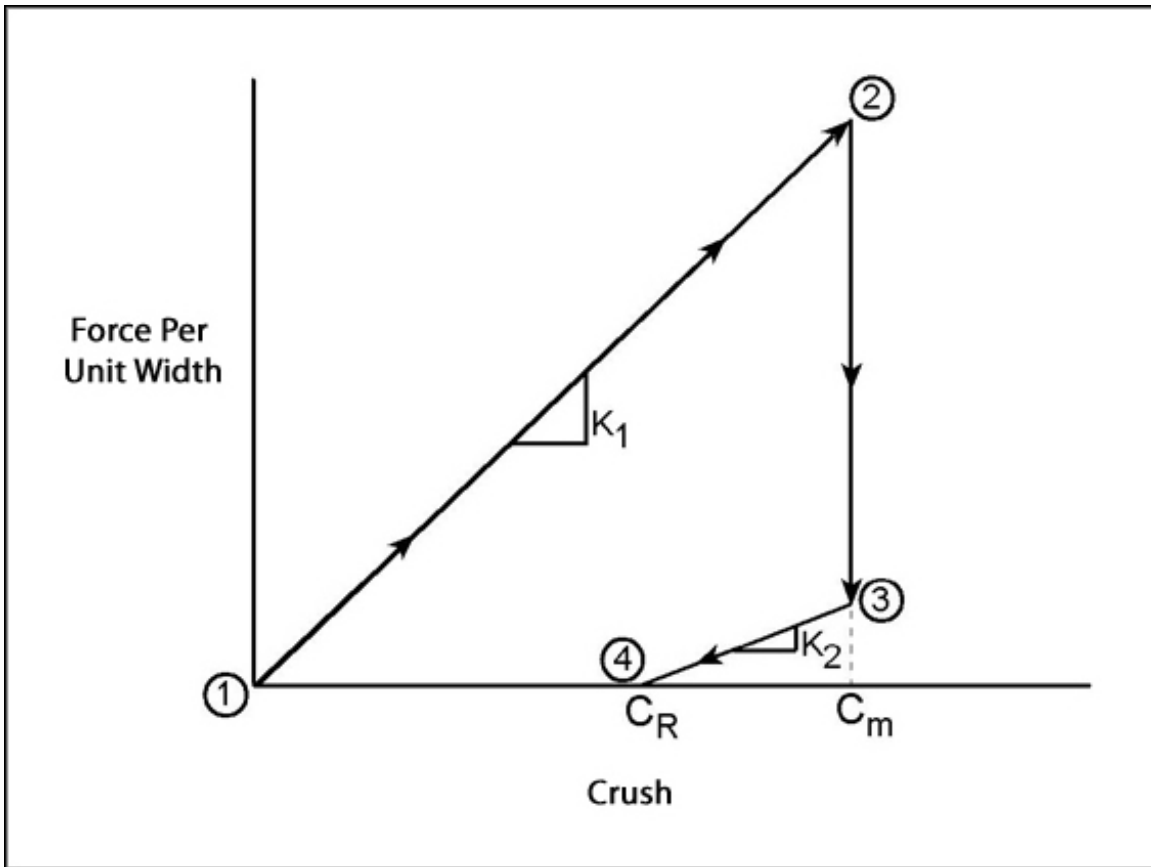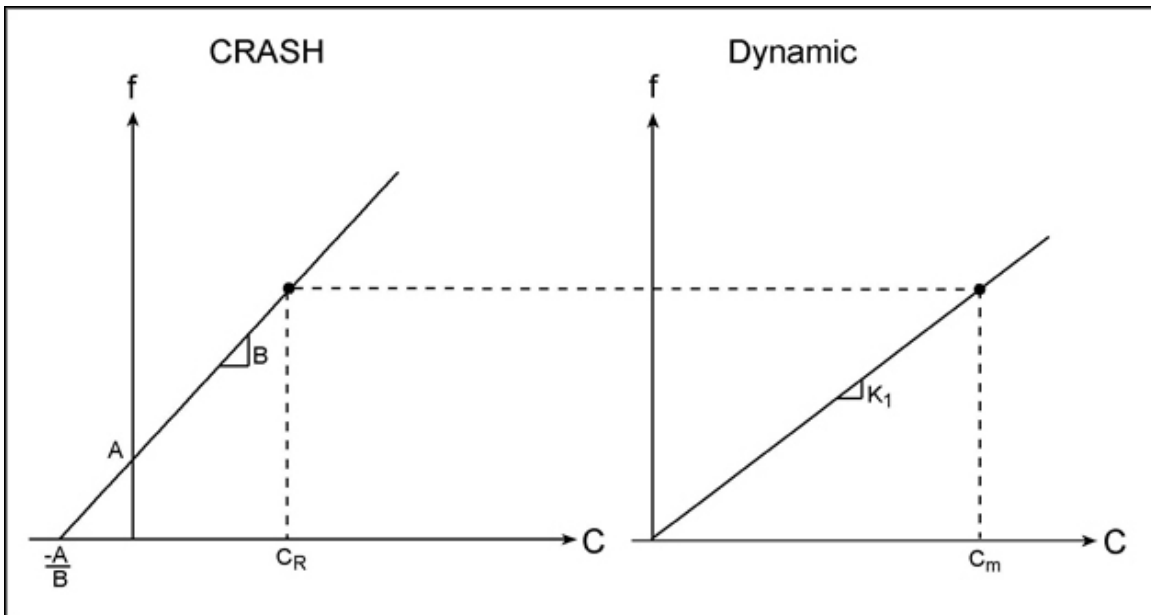
Figure 1 – Idealized Dynamic Force-Crush Curve

Figure 2 – CRASH and Dynamic Force-Crush Models
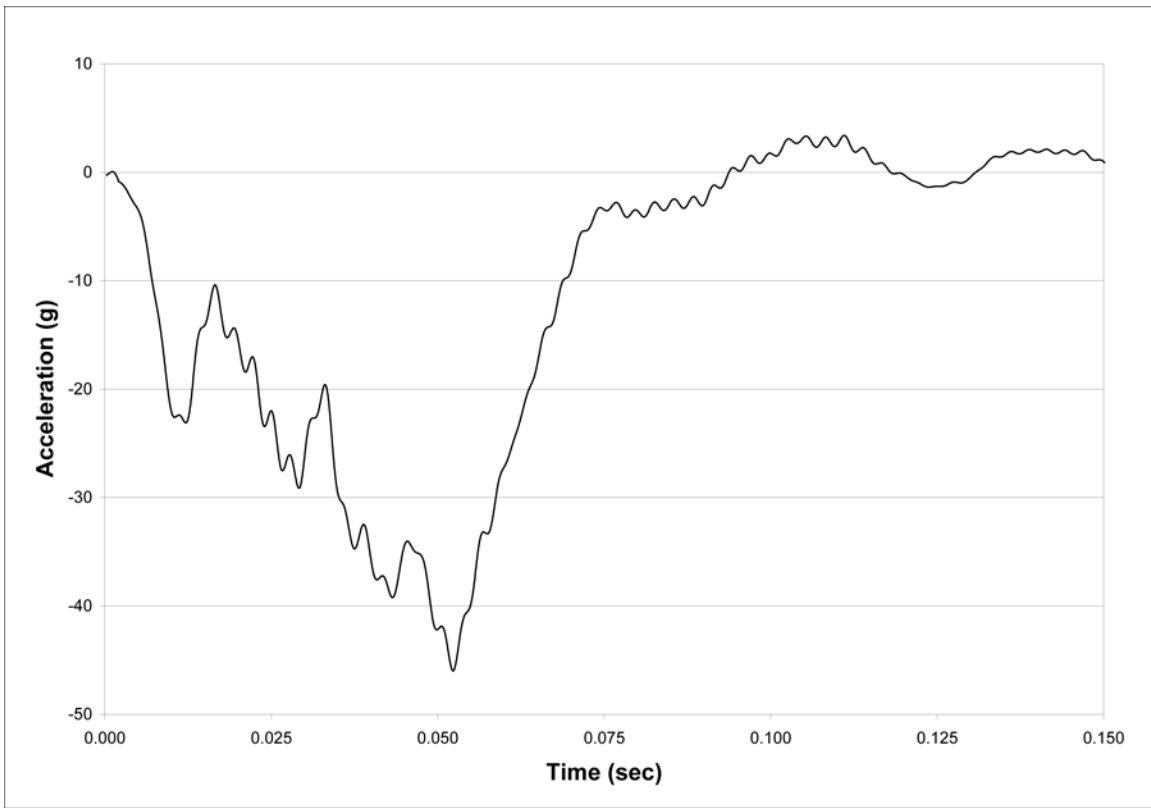
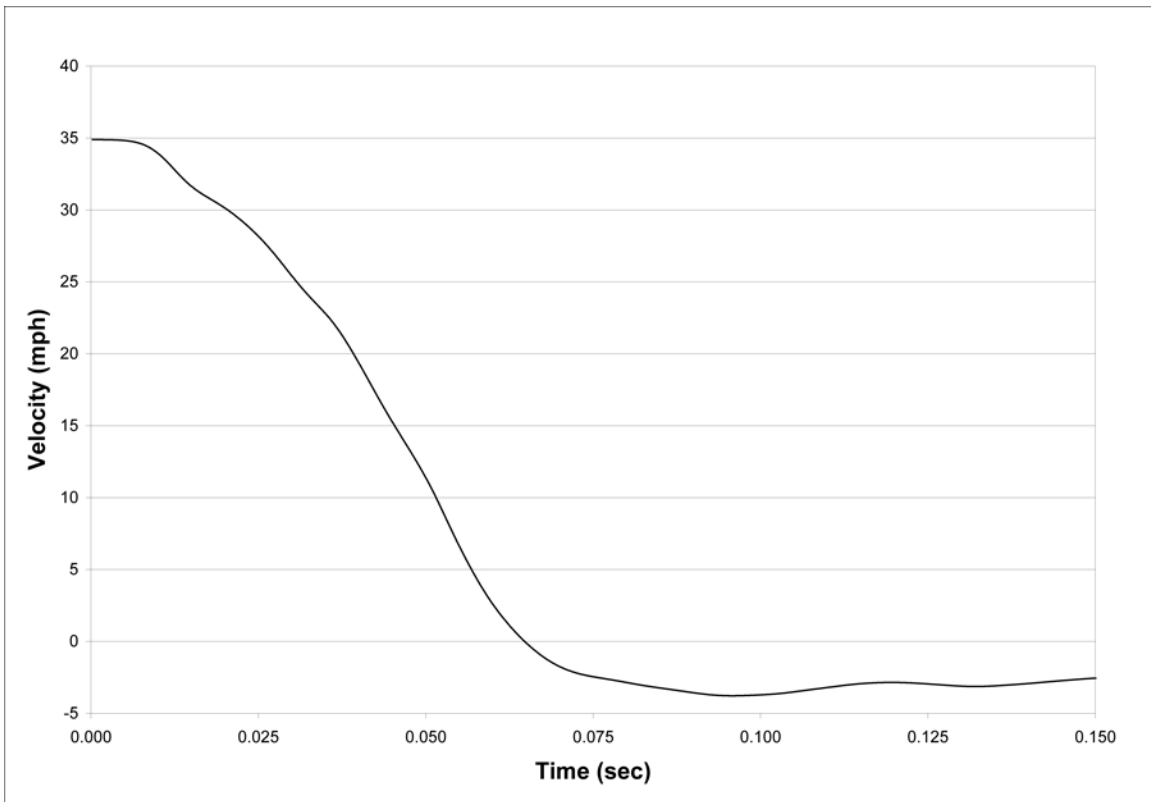**Figure 3** – Acceleration v. Time Curve for NHTSA Test #800

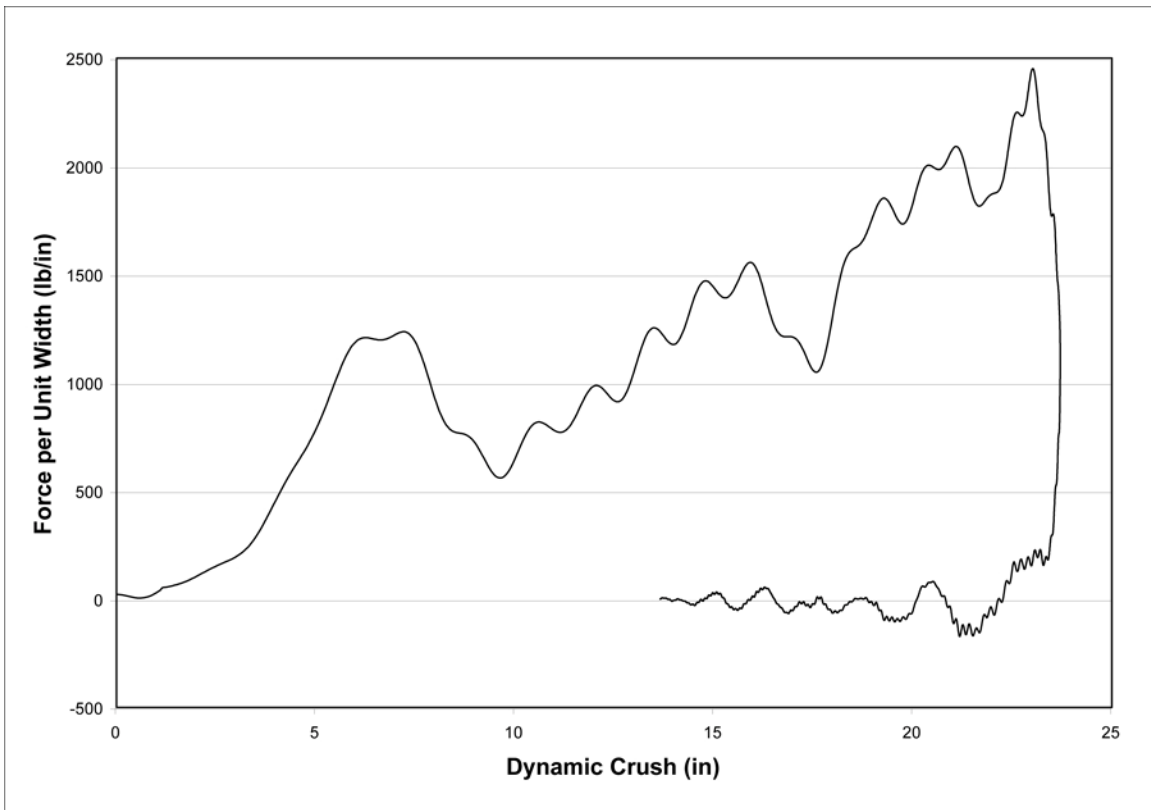**Figure 4** – Velocity v. Time Curve for NHTSA Test #800

Figure 5 – Force-Crush Curve for NHTSA Test #800
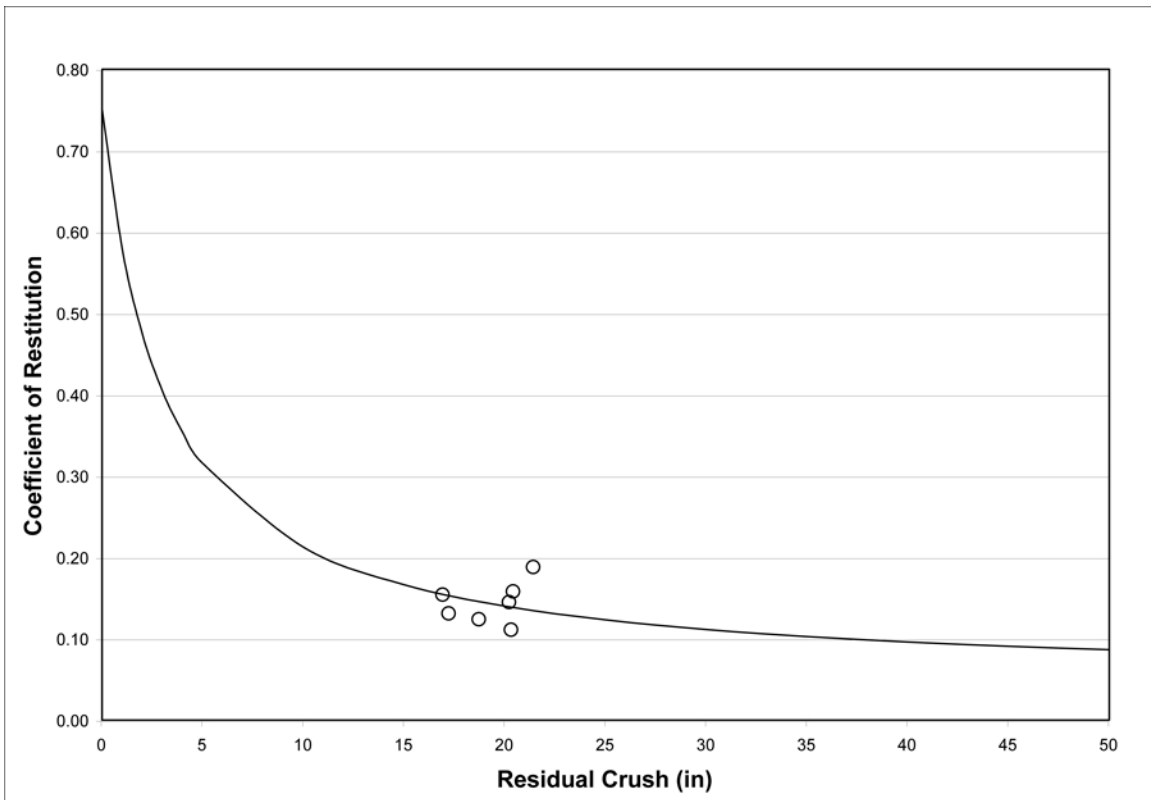

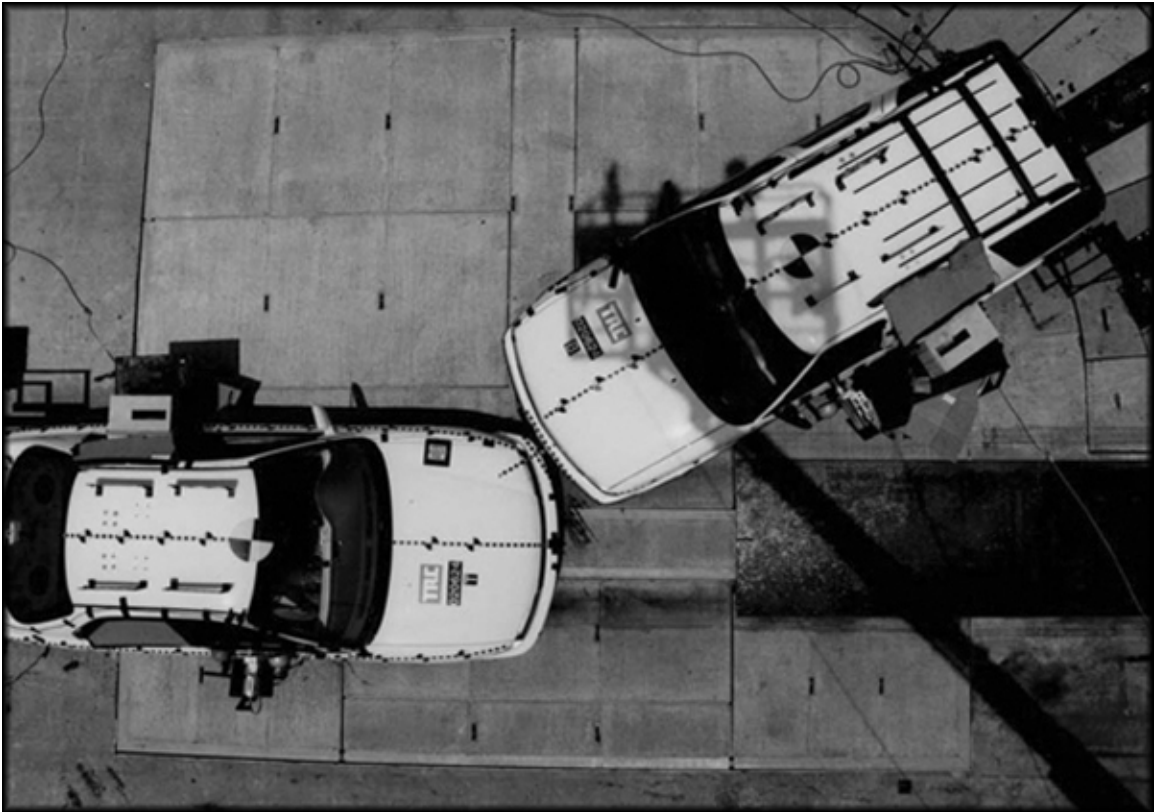**Figure 6** – Equation (13) with ε_{max} = 0.75 and ζ = 0.94

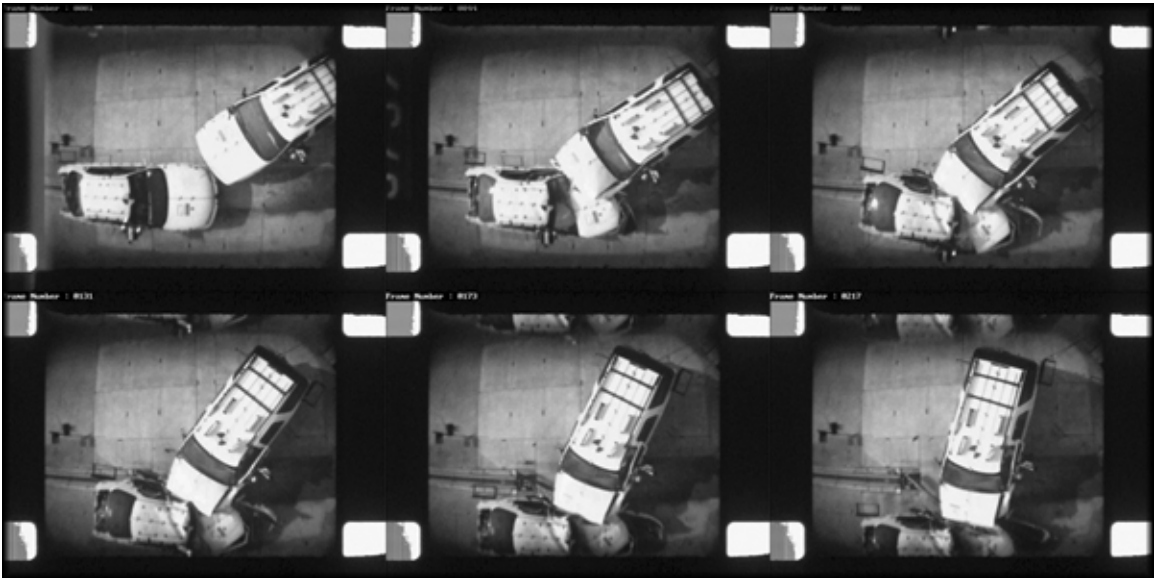**Figure 7** – Impact Configuration (Test #4363)

Figure 8 – Collision Dynamics (Test #4363)
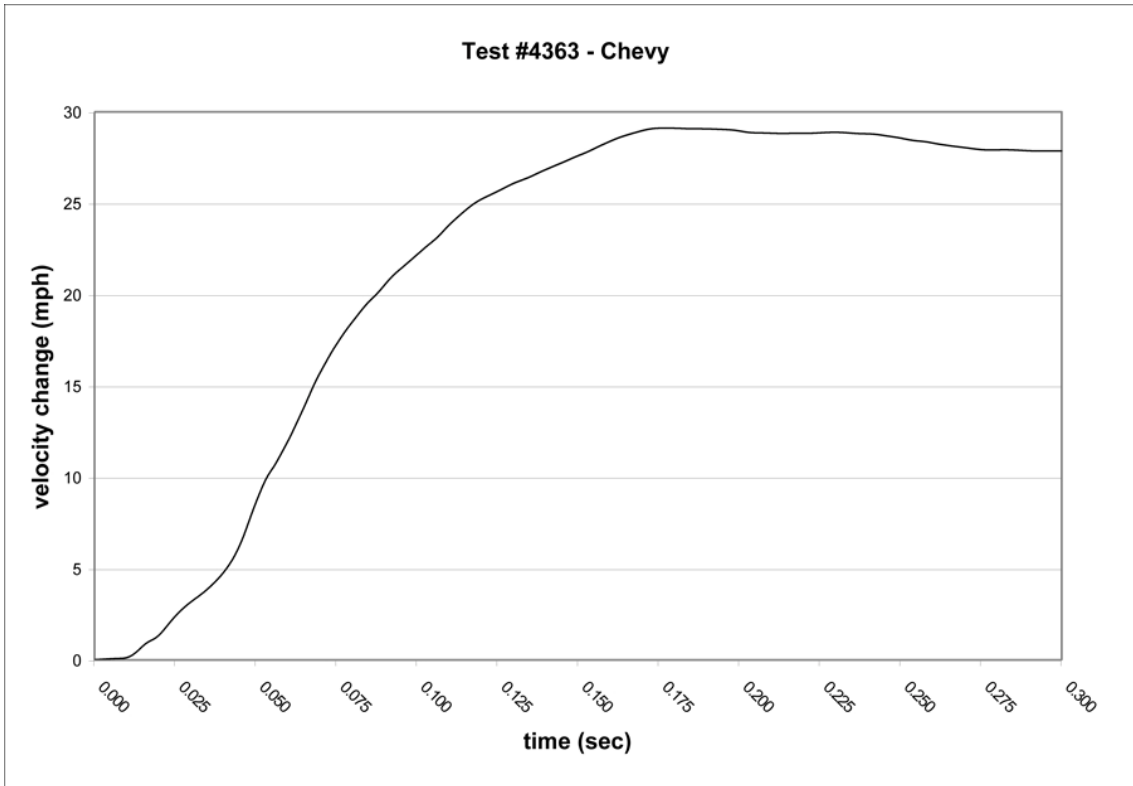
Figure 9 – ΔV Plot for the Blazer (Test #4363)
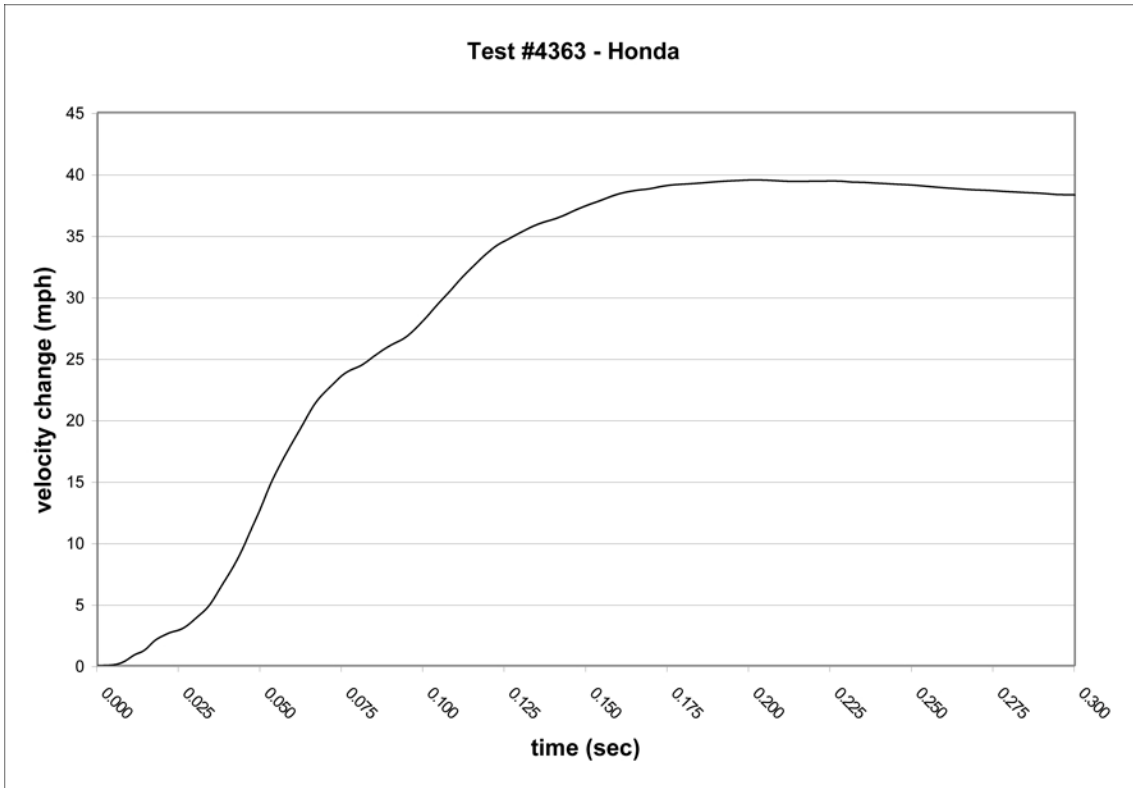
Figure 10 – ΔV Plot for the Accord (Test #4363)
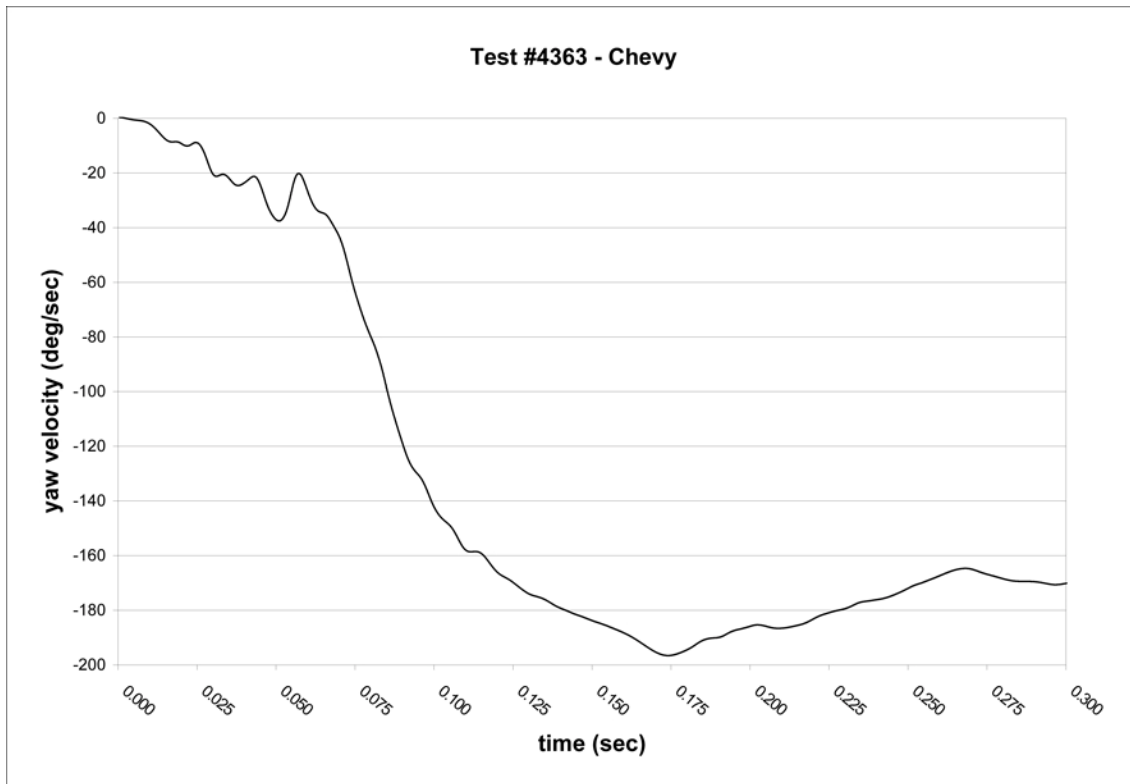
Figure 11 – Yaw Velocity Plot for the Blazer (Test #4363)
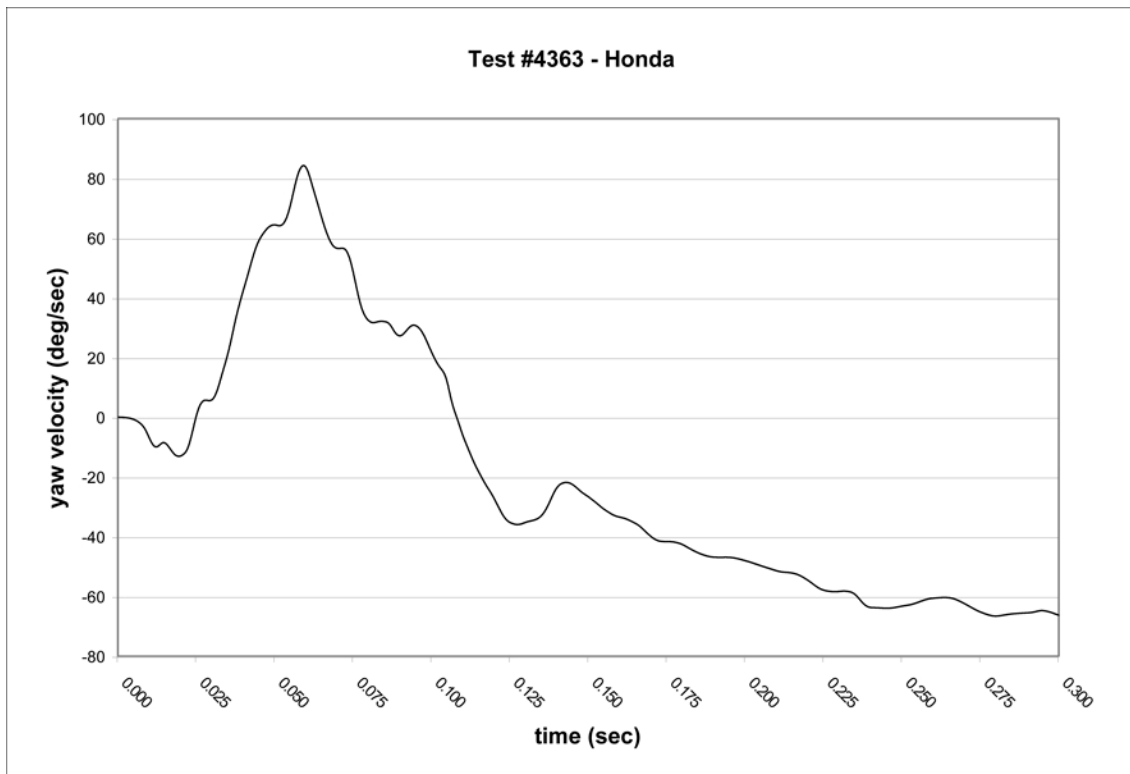
Figure 12 – Yaw Velocity Plot for the Accord (Test #4363)[19]

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[19] The accelerometer data indicates the Accord initially reaches a peak yaw rate of about 75 degrees per second in the opposite direction from that which it eventually ends up rotating. Review of the crash test video reveals that this apparent rotation early in the impact is due largely to vehicle deformation and inertial effects. By the time of separation, the effects of deformation and inertia have dissipated and the yaw velocity is a more accurate reflection of the true yaw rate.

Honda	Inertial and Geometric Parameters	Chevy
3451	Weight (lb)	4695
2152	Yaw Moment of Inertia (lb-ft-sec^2)	2804
2.6	Distance from CG to Impact Center (IC) (ft)	5.7
23.8	Angle Between Heading and CG-IC Line (deg)	10.6
180	Heading Angle at Impact (deg)	32

	Initial Velocities	
35.1	Initial Translational Velocity (mph)	35.0
0.0	Initial Yaw Velocity (deg/s)	0.0

	Final Velocities (mph)	
-0.1	V$_x$	4.0
18.5	V$_y$	4.9
18.5	V	6.4

31	Calculated Post-Impact Yaw Velocity (deg/s)	182
31-80	Post-Impact Yaw Velocity from Accelerometer Data (deg/s)	173-239

39.5	Calculated Change in Velocity (mph)	29.1
39.5	Change in Velocity from Accelerometer Data	29.1

27.7	Inertial Frame PDOF (deg)	

Impact Parameters	
Coefficient of Restitution	0.126
Critical Impulse Ratio	-0.51
Impulse Ratio (% of Critical)	100%
Impulse Ratio	-0.51
Orientation of Normal Direction (deg)	55

System Kinetic Energy (ft-lb)	
Initial	334277
Final	60360
Loss	273917
Percent Energy Loss	81.9%
Normal Direction Energy Loss	185759
Tangential Direction Energy Loss	88158

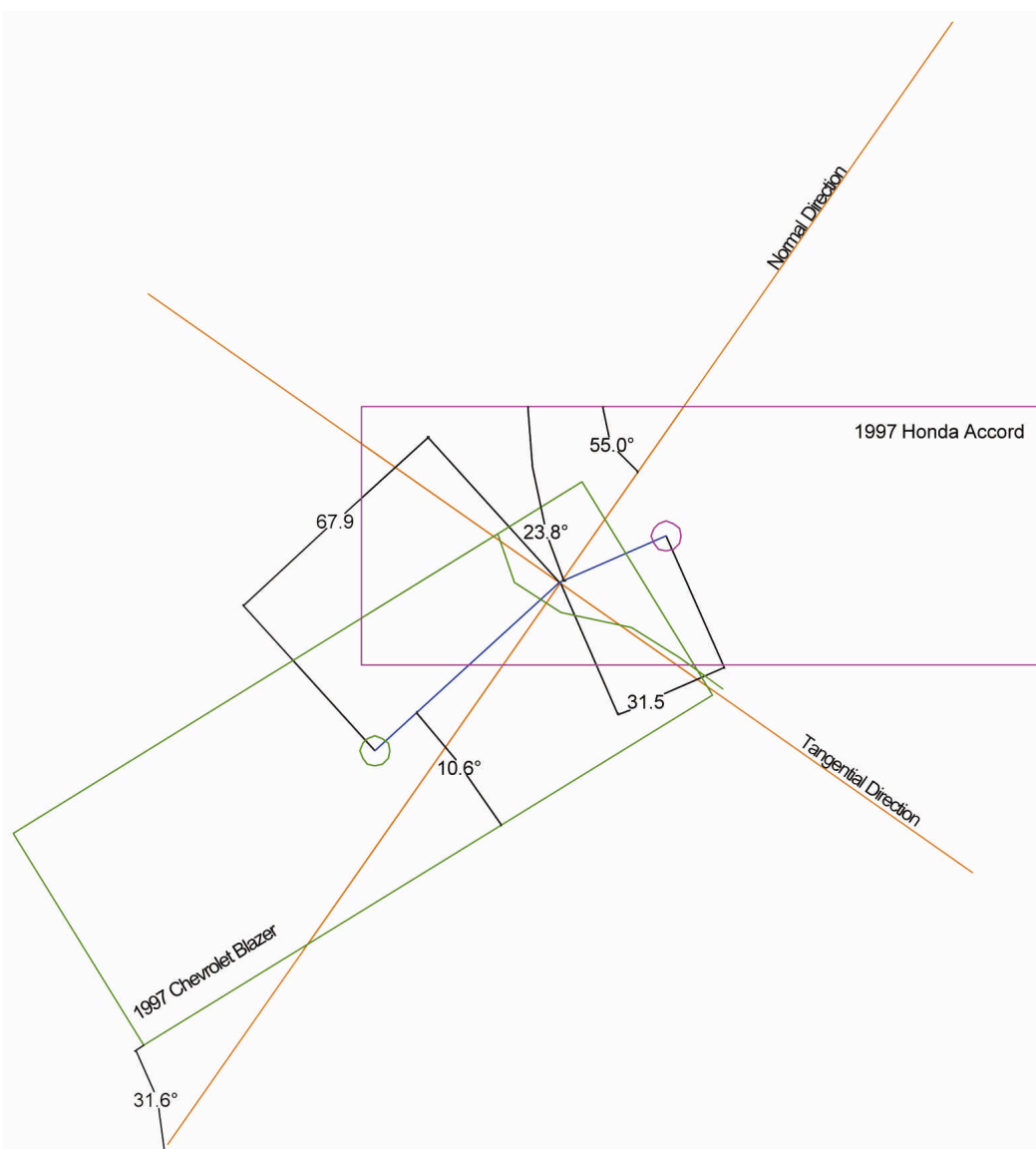**Figure 13** – Planar Impact Mechanics Calculations (Test #4363)

Figure 14 – Contact Surface and Impact Center Location (Test #4363)

	1997 Chevrolet Blazer				Crush (in)	A (lb/in)	B (lb/in2)	Width (in)	Force (lb)	Absorbed Energy (ft-lb)
Total Vehicle Weight	4695	lb	C_1	30.3						
Mass	145.8	lb-s²/ft	Zone 1			304.3	89.5	11	31947	43322
Yaw Moment of Inertia	2804	lb-ft-s²	C_2	27.8						
Radius of Gyration	4.4	ft	Zone 2			304.3	89.5	11	29043	35825
			C_3	24.4						
Collision Force Moment Arm	17.0	in	Zone 3			304.3	89.5	11	26188	29115
Effective Mass Multiplier, g	0.91		C_4	22.0						
Adjusted Mass	132.0	(lb*s^2)/ft	Zone 4			304.3	89.5	11	18558	15199
Direction of Collision Force	4	deg	C_5	8.9						
Deformation Location (F, B, L, R)	F		Zone 5			304.3	89.5	11	9451	3890
Oblique Impact Correction Factor	1.00		C_6	3.5						
Maximum Coefficient of Restitution	0.62		Zone 6			304.3	89.5	0	0	0
b_0	4.48	mph	C_7	0.0						
zeta	0.87		Zone 7			304.3	89.5	0	0	0
			C_8	0.0						
			Zone 8			304.3	89.5	0	0	0
			C_9	0.0						

Total Damage Width	55
Total Force	115187
Force Ratio	1.00
Total Absorbed Energy	127351
EBS (mph)	29.9
Vehicle Rest. Coeff.	0.164

Figure 15 – Crush Analysis Input Worksheet (Blazer, Test #4363)

	1997 Honda Accord				Crush (in)	A (lb/in)	B (lb/in²)	Width (in)	Force (lb)	Absorbed Energy (ft-lb)
Total Vehicle Weight	3451	lb	C_1	27.1						
Mass	107.2	lb-s²/ft	Zone 1			265.0	105.0	12	39090	57268
Yaw Moment of Inertia	2152	lb-ft-s²	C_2	29.9						
Radius of Gyration	4.5	ft	Zone 2			265.0	105.0	12	36822	50984
			C_3	23.5						
Collision Force Moment Arm	2.2	in	Zone 3			265.0	105.0	12	23718	22096
Effective Mass Multiplier, g	1.00		C_4	9.1						
Adjusted Mass	107.0	(lb*s^2)/ft	Zone 4			265.0	105.0	12	10614	4422
Direction of Collision Force	-28	deg	C_5	2.7						
Deformation Location (F, B, L, R)	F		Zone 5			265.0	105.0	12	4881	928
Oblique Impact Correction Factor	1.13		C_6	0.0						
Maximum Coefficient of Restitution	0.66		Zone 6			265.0	105.0	0	0	0
b_0	4.10	mph	C_7	0.0						
zeta	0.85		Zone 7			265.0	105.0	0	0	0
			C_8	0.0						
			Zone 8			265.0	105.0	0	0	0
			C_9	0.0						

Total Damage Width	60
Total Force	115125
Force Ratio	1.00
Total Absorbed Energy	135698
EBS (mph)	34.3
Vehicle Rest. Coeff.	0.165

Figure 16 – Crush Analysis Input Worksheet (Accord, Test #4363)

Outputs		
	Chevy	Honda
Overall Coefficient of Restitution		
Energy Equivalent	0.164	
Mass Equivalent	0.165	
Stiffness Equivalent	0.164	
Average	0.164	
Impact Force Ratio	1.00	
Total Absorbed Energy (ft-lb)	263049	
Total Dissipated Energy (ft-lb)	255932	
Approach Change in Velocity (mph)	26.1	35.5
Total Change in Velocity (mph)	30.4	41.3
Velocity Changes Calculated with Average Coefficient of Restitution		

Figure 17 – Crush Analysis Output Sheet (Test #4363)

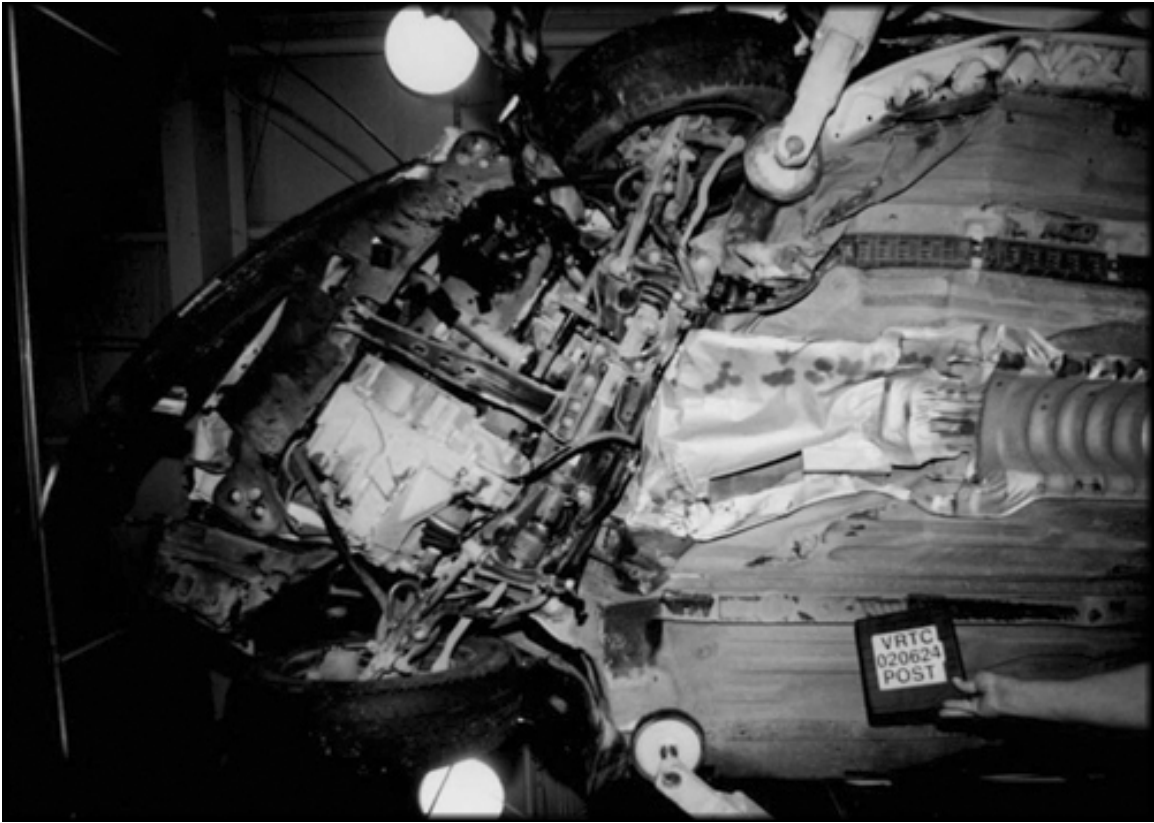**Figure 18** – Blazer Undercarriage, Post-Test (Test #4363)
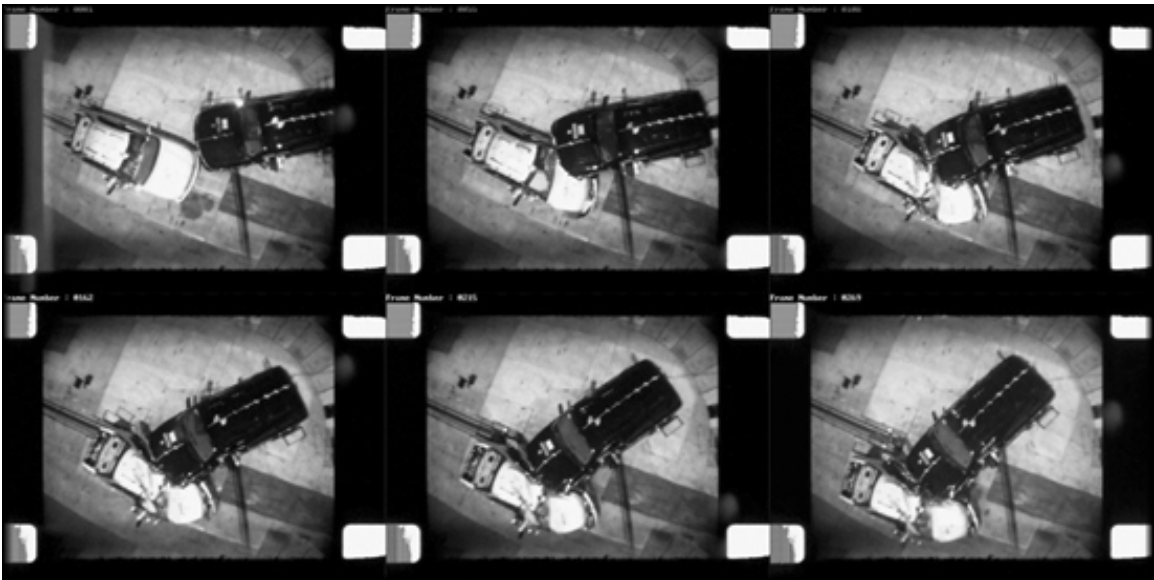
Figure 19 – Accord Undercarriage, Post-Test (Test #4363)

Figure 20 – Collision Dynamics (Test #4364)

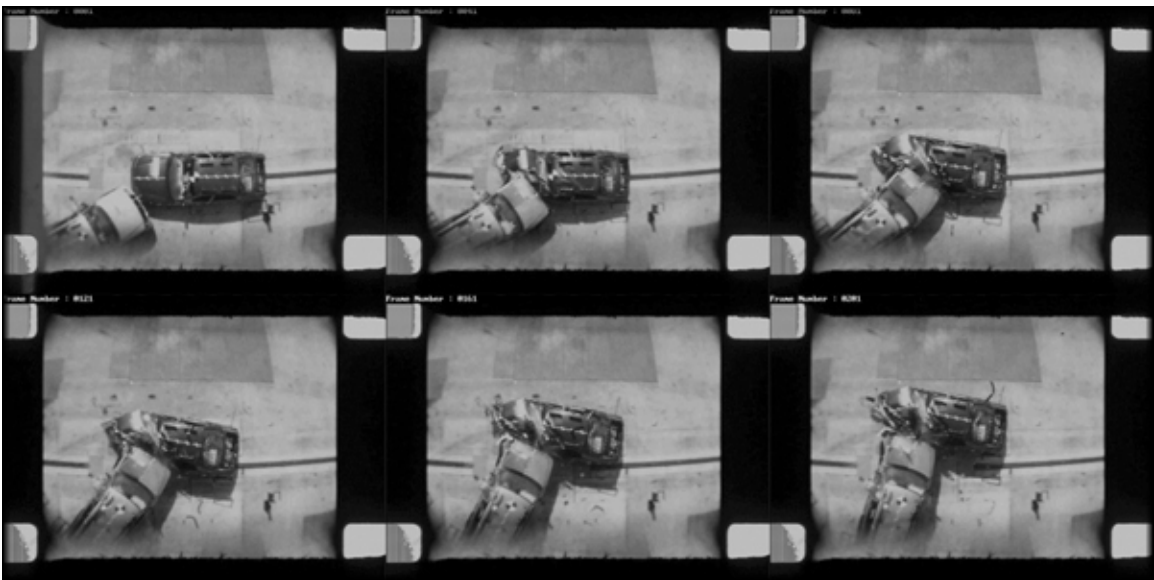**Figure 21** – Collision Dynamics (Test #4474)

Figure 22 – Collision Dynamics (Test #4438)

TABLES

Test #	Initial Speed, mph (kph)	ε	C_R/C_m	ε_{max}	ζ
800	34.8 (56.0)	0.111	0.83	0.65	0.97
1677	35.0 (56.3)	0.158	0.82	0.86	0.94
1692	29.5 (47.5)	0.131	0.82	0.73	0.97
1979	34.9 (56.2)	0.145	0.82	0.79	0.94
2046	29.3 (47.2)	0.124	0.82	0.69	0.96
2071	29.4 (47.3)	0.154	0.79	0.74	0.94
2404	35.2 (56.6)	0.188	0.76	0.79	0.87
Average				**0.75**	**0.94**

Table 1 – ε_{max} and ζ Data for Chevrolet Astro Van Frontal Impact Crash Tests

Test #	Actual ΔVs (mph)	Calculated ΔVs (mph)	% Difference	Actual ε	Calculated ε	% Difference
4363	29.1, 39.5	28.3, 38.5	-2.6% (-17.0%)	0.126	0.170	34.9%
4364	26.6, 40.1	26.6, 40.0	-0.1% (-13.0%)	0.089	0.148	66.3%
4438	27.6, 39.9	28.4, 40.5	2.8%, 1.6% (-10.6%, -11.5%)	0.095	0.148	55.8%
4474	28.5, 40.7	25.8, 36.3	-9.35%, -10.75% (-21.6%, -22.8%)	0.150	0.157	4.7%

Table 2 – Validation Study Results: Part I

Test #	4363	4364	4438	4474
Collision Geometry	Front-to-Left Front, 50% Offset, 30 Degrees	Front-to-Left Front, 50% Offset, 30 Degrees	Front-to-Left Front, 50% Offset, 30 Degrees	Front-to-Left Front, 50% Offset, 30 Degrees
Vehicles	Chevrolet Blazer, Honda Accord	Chevrolet Trailblazer, Honda Accord	Mitsubishi Montero, Honda Accord	Mitsubishi Montero, Honda Accord
Initial System Energy (ft-lb)	334277	354657	326149	326559
Collision Total Dissipated Energy (ft-lb)	273917	282851	267960	257536
Collision Total Dissipated Energy (%)	81.9	79.8	82.2	78.9
Normal Direction Energy Loss, Crushing (ft-lb)	185759	210416	198815	191632
Tangential Direction Energy Loss, Sliding (ft-lb)	88158	72434	69145	65904
Coefficient of Restitution	0.126	0.089	0.095	0.161
Impulse Ratio	-0.51	-0.42	-0.44	-0.40
Crush Analysis (Woolley Correction)				
Calculated Dissipated Crush Energy (ft-lb)	219695	247983	237740	191403
Difference w/ Normal Direction Energy Loss (%)	18.3%	17.9%	19.6%	-0.1%
Difference w/ Total Dissipated Energy (%)	-19.8%	-12.3%	-11.3%	-25.7%
Calculated Coefficient of Restitution	0.170	0.148	0.148	0.157
Crush Analysis (CRASH3 Correction)				
Dissipated Crush Energy (ft-lb)	235301	284572	268000	221584
Difference w/ Normal Direction Energy Loss (%)	26.7%	35.2%	34.8%	15.2%
Difference w/ Total Dissipated Energy (%)	-14.1	0.6%	0.0%	-14.2%
Calculated Coefficient of Restitution	0.168	0.144	0.145	0.152

Table 3 – Validation Study Results: Part II