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Speed Analysis of Yawing Passenger Vehicles Following a Tire Tread Detachment

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Abstract

This paper presents yaw testing of vehicles with tread removed from tires at various locations. A 2004 Chevrolet Malibu and a 2003 Ford Expedition were included in the test series. The vehicles were accelerated up to speed and a large steering input was made to induce yaw. Speed at the beginning of the tire mark evidence varied between 33 mph and 73 mph. Both vehicles were instrumented to record over the ground speed, steering angle, yaw angle and in some tests, wheel speeds. The tire marks on the roadway were surveyed and photographed.

The Critical Speed Formula has long been used by accident reconstructionists for estimating a vehicle's speed at the beginning of yaw tire marks. The method has been validated by previous researchers to calculate the speed of a vehicle with four intact tires. This research extends the Critical Speed Formula to include yawing vehicles following a tread detachment event. The Critical Speed Formula was found to produce results of acceptable and known accuracy, provided the appropriate inputs are used for the given situation and several guidelines are observed. The inputs and guidelines for the use of the Critical Speed Formula for these tread detachment scenarios are discussed.

For all tests analyzed, the tire mark evidence was documented with survey equipment, photographs and drone footage. In the past, it may have been necessary to take tire mark radius measurements in the field for use in the Critical Speed Formula. However, with the advent of modern documentation techniques, radius measurements can be taken from a scaled scene diagram and acceptable accuracy in the speed calculations can be achieved.

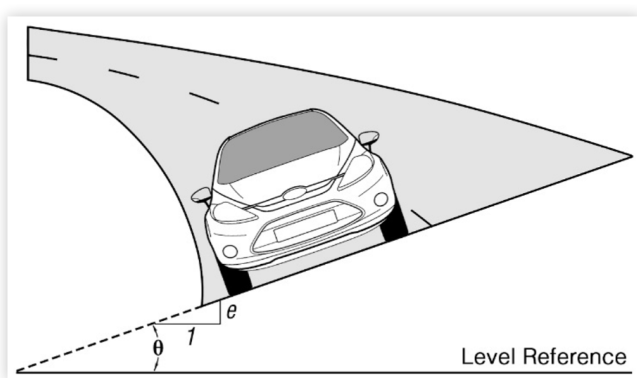
Introduction

During a single vehicle loss of control event, it is common for the driver to steer the vehicle into a yaw. A yawing vehicle develops a sideslip angle, or a discrepancy between the heading direction and the travel direction. Curved tire marks called yaw marks are often deposited by a yawing vehicle. The Critical Speed Formula is used by accident reconstructionists to determine the speed of a yawing vehicle from the curved path the vehicle follows, as established by the yaw marks on the road. The Critical Speed Formula has been studied in the literature for decades. The 2017 SAE Recommended Practice J2969, Use of the Critical Speed Formula, "provides guidelines for procedures and practices used to obtain and record measurements and to analyze the results of the critical speed method." [1] The recommended practice considered many publications, most of which support the use of the Critical Speed Formula and a few that are critical of the method [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. The Critical Speed Task Force, who authored J2969, determined that the method is accurate within -13.5% (underestimate in speed) to +10% (overestimate in speed) when used correctly. The authors mention that the accuracy of the results can be improved if corrections are made for braking, coasting or accelerating conditions. The recommendations of J2969 are summarized below.

The inputs to the Critical Speed Formula (Equation 1) include the equivalent tire roadway friction coefficient (μ_{eq}), the radius (r), the superelevation of the road (e), and the acceleration due to gravity (g). Figure 1 depicts a car traveling a banked curved road with superelevation.

$$v = \sqrt{\frac{gr\left(\mu_{eq} + e\right)}{\left(1 - \mu_{eq} e\right)}} \tag{1}$$

FIGURE 1 A car traveling on a curved road with superelevation (Figure 1 from J2969).



On nearly level surfaces, Equation 1 reduces to:

$$v = \sqrt{g r \mu_{eq}} \tag{2}$$

The friction of the roadway is to be measured in accordance with SAE Recommended Practice J2505, Measurement of Vehicle-Roadway Frictional Drag [28]. J2505 recommends conducting skid to stop tests with ABS disabled in an instrumented vehicle. Half of the build-up and half of the drop-off of acceleration are included in the calculation of friction. Figure 2 (Figure 1 from J2505) depicts how the friction coefficient is calculated from the filtered data.

In accordance with J2969, the radius is measured from the front outside tire mark. There should be at least two curved tire marks, from the outer tires, and the rear tire should be tracking outside the front tire. Typically, the tire marks contain evidence of diagonal striations consistent with a yawing vehicle [29,30,31]. The radius of the front outer mark can be calculated using measurements of the chord and middle ordinate. The measurement should be made from the first visible evidence of critical speed, where the tire marks begin to diverge or off-track. For higher speeds, the chord of at least 15 m (~50 feet) should be used, longer if necessary to achieve a middle ordinate of at least 0.15 m (6 inches). Measurements should be taken from the outer edge of the mark. Over the segment of measured chord, the separation of the tire marks should be less than half the track width of the vehicle. The radius (r) can be computed from the chord (l) and middle ordinate (h) with Equation 3.

$$r = \frac{l^2}{8h} + \frac{h}{2} \tag{3}$$

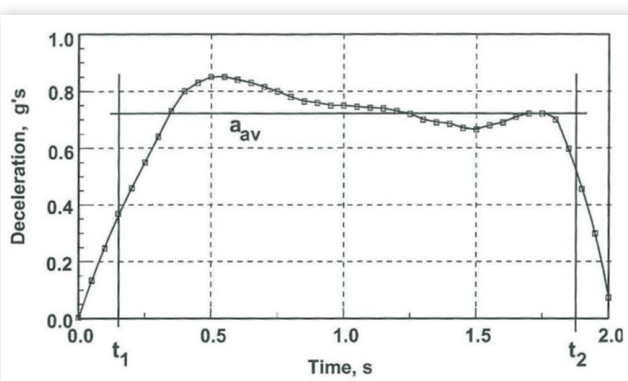
Brach and Brach included a chapter on Critical Speed in their 2011 textbook [9]. Included in the chapter was a discussion on the accuracy differences in the method depending on whether the driver was braking, coasting, or accelerating. They found the differences to be significant, and reported the following averages, all underestimates of actual speed:

Braking: -13.5%

Coasting: -4.6%

Acceleration: -1.2%

FIGURE 2 Acceleration time history from a skid to stop test. Acceleration from t_1 to t_2 is used to calculate the roadway friction coefficient. (Figure 1 from SAE J2505)



Brach also reported a set of guidelines for using the Critical Speed Formula based on guidelines first reported by Lambourn [19]. Those guidelines are repeated below verbatim [9]:

1. “There should be at least two tire marks visible, they should be from the outside wheels, and they should show lateral striations or scratches. There should be clear evidence that the rear wheels were tracking outside the front wheels.
2. The measurement of the radius should be made from the front outside mark (leading front tire).
3. The measurement of the chord should be made at the earliest point corresponding to Guideline 1.
4. A chord length of about 15 m (about 50 ft) is suitable but a longer chord should be taken when the middle ordinate is less than 0.3 m (1 ft) to minimize measurement errors.
5. The separation of the front and rear tire marks over the length of the measured chord should be no more than about one half of the track width (although they may diverge more along the marks).”

This research expands the Critical Speed Formula to include the case of tire tread detachment. Testing for this study was conducted with two vehicles over several years at multiple facilities. Modifications were made to the Critical Speed Formula to account for a tire without tread. Those modifications were based on theoretical and empirical analysis and will be discussed below.

Test Sites

The Denver Police Training Center was used to conduct yaw testing of the Chevrolet Malibu. The testing surface is asphalt and was dry and free of irregularities at the time of all testing. The dimensions of the rectangular testing surface are roughly 700 by 300 feet. The test location had a superelevation of approximately -0.6 degrees along the vehicle’s path of travel. Testing at this facility was conducted on June 24, 2011. Figure 3, an image from Google, depicts the facility.

Testing of the Ford Expedition was conducted on May 31, 2012, at Front Range Airport, near Denver, Colorado. The test site has a superelevation of +0.6 degrees along the test vehicle’s

FIGURE 3 Denver Police Training Center Site.

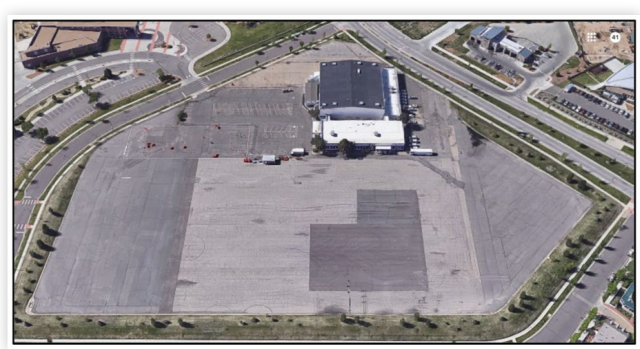


FIGURE 4 Testing grounds at Front Range Airport.

FIGURE 5 EVOC Test Facility.

path of travel and is free of any pavement defects. The surface was dry at the time of testing and measures 1200 by 400 feet. Figure 4, an image from Google, depicts the test site.

The Douglas County Emergency Vehicle Operation Center (EVOC) was used to conduct testing of the Ford Expedition and Chevrolet Malibu. The test facility includes a test track and several skid pads. The larger of the two skid pads was used and measures 800 by 500 feet. The test pad was generally flat and level, and free of any pavement defects. Testing was conducted on July 24, 2018 and September 28, 2018. Figure 5 depicts the facility.[1] The pavement was dry at the time of testing.

Test Vehicles

Two vehicles from different manufacturers were used in the testing. These vehicles included a passenger car and a Sport Utility Vehicle (SUV). The vehicles were retrofit with a roll cage and five-point safety harnesses.

Chevrolet Malibu

The test vehicle was a four-door, 2004 model year Chevrolet Malibu LT (VIN - 1G1ZU54854F135916). The vehicle was equipped with a 3.5-liter, 6-cylinder gasoline engine and a four-speed, front-wheel-drive automatic transmission. Safety features include four-wheel anti-lock brakes and traction control. The Malibu has an electric, power-assisted variable-speed rack and pinion steering, and independent front and rear suspension. The vehicle was outfitted with 225/60R16 tires during early testing. In later testing, 215/60R16 tires were used, which match the original equipment size specification. At the time of testing, the vehicle with instrumentation and driver weighed between 3262 puonds and 3449 pounds on the different test dates. The front axle weight distribution was between 62% and 64%. Figure 6 depicts the Malibu.

FIGURE 6 Test Chevrolet Malibu.

FIGURE 7 Test Ford Expedition.

Ford Expedition

The 2003 model year Expedition XLT (VIN - 1FMRU15W23LC49770) was equipped with a 4.6-liter, 8-cylinder gasoline engine and a four-speed, rear-wheel-drive automatic transmission. The Expedition has independent front and rear suspension. At the time of the 2012 and 2018 testing, the vehicle weighed 5,610 and 5,560 pounds including instrumentation and driver respectively, with a 49% front weight distribution. The tire size equipped for all testing was the OEM specified 265/70-R17. Outriggers were attached to the Expedition. Figure 7 depicts the Expedition.

Tire Preparation

A variety of tires from different manufacturers were used in the testing. Cuts were made to the tires so that the tread and

[1] http://hrletf.org/evoc/

FIGURE 8 Tire with tread and top belt removed.

top belt could be removed. First, a single cut was made across the tread along the belt bias. This cut went through top belts. The shoulder on both sides of the tire was then cut around the entire circumference. The tread and top belt were then pulled off the tire. Figure 8 depicts a tire prepared to be tested with the Expedition.

Full Scale Testing

Instrumentation

The test vehicles were instrumented with Racelogic VBOX VB20SL3 data acquisition equipment. This device recorded Slip, Pitch, and Roll angles in addition to the vehicle's speed, during each test. The Malibu's CAN Interface was integrated into the VBOX system and recorded wheel speed and steering position from the vehicle communication network.

During the testing, multiple cameras were used to document the vehicle's motion. For the 2018 testing, in addition to several externally and internally mounted vehicle cameras, and stationary external cameras, drone cameras were used to obtain aerial documentation of the testing. Ground control points were placed and surveyed at the test site.

Pretest

Prior to running any tire disablement tests, the vehicles were weighed and photographed. Slow speed straight runs were conducted to align the GPS antenna to the vehicle's heading and zero out any static roll. Steering sensors were calibrated. A pre-test drive was conducted to test the functionality of all instrumentation and cameras.

Test Protocol

The vehicles were setup for yaw testing in four different configurations; four tires intact, a prepared tire in the outside front corner position, a prepared tire in the outside rear corner position, and a prepared tire in the inside rear corner position. The outside tire is on the leading side, and the inside tire is on the trailing side. For example, if the vehicle yawed counterclockwise, in a right side leading orientation, the right side tires were the outside tires. The vehicle was accelerated up to test speed and the accelerator was released. In most tests, the driver made and held a single large steering input in one direction. In some tests, two steering inputs were made prior to the final yaw, one in each direction. The brakes were not applied until the end of the tests.

Following the yaw tests, ABS-disabled skid-to-stop tests were conducted to determine the tire/roadway coefficient of friction. Two different vehicle configurations were tested. Four tires with tread intact were placed on the vehicle to determine the friction coefficient of the unprepared tires. The vehicles were then equipped with four tires, all with the tread removed, to determine the friction coefficient of the tires without tread.

A test matrix for each vehicle and day is shown in Table 1 and 2. The driving was performed by Gray Beauchamp, David Pentecost, Daniel Koch, and William Bortles. On July 24, 2018, testing of the Malibu was cut short due to weather.

Testing Results

Seven yaw tests were conducted on June 24, 2011 at the Denver Police Training Center, ten yaw tests on May 31, 2012 at the

TABLE 1 Malibu test matrix - number of each test.

6/24/2011	Yaw - Four Unaltered Tires	2
	Yaw - Rear Outside Tire Tread Removed	3
	Yaw - Rear Inside Tire Tread Removed	2
	Skid to Stop - Four Unaltered Tires	3
7/24/2018	Yaw - Four Unaltered Tires	3
9/28/2018	Yaw - Four Unaltered Tires	3
	Yaw - Front Outside Tire Tread Removed	3
	Yaw - Rear Outside Tire Tread Removed	3
	Yaw - Rear Inside Tire Tread Removed	3
	Skid to Stop - Four Unaltered Tires	3
	Skid to Stop - Four Tires Tread Removed	1

TABLE 2 Expedition test matrix - number of each test.

5/31/2012	Yaw - Four Unaltered Tires	3
	Yaw - Front Outside Tire Tread Removed	3
	Yaw - Rear Outside Tire Tread Removed	4
	Skid to Stop - Four Unaltered Tires	3
	Skid to Stop - Four Tires Tread Removed	3
7/24/2018	Yaw - Four Unaltered Tires	3
	Yaw - Front Outside Tire Tread Removed	3
	Yaw - Rear Outside Tire Tread Removed	3
	Skid to Stop - Four Unaltered Tires	4
	Skid to Stop - Four Tires Tread Removed	2

Front Range Airport, twelve yaw tests on July 24, 2018 at the EVOC, and twelve yaw tests on September 28th, 2018 at the EVOC. In all, 41 yaw tests were conducted with various tire configurations. The range of speeds for the tests was 33 to 73 mph at the beginning of the analyzed tire mark evidence.

General differences were noted among the different tire conditions when large steering inputs were made at speed. Vehicles with a tire with the tread removed placed at a front outer position have the tendency towards a shallower path, or more understeer, compared to a vehicle with four unaltered tires. Vehicles with a tire with the tread removed placed at a rear outer position have the tendency towards a sharper path, or more oversteer, compared to a vehicle with four unaltered tires. Vehicles with an outer rear altered tire spun out during the testing. These trends have been noted by other authors [32,33]. During the higher speed Malibu testing (approximately 60 mph or greater), the vehicle spun out during the tests with four unaltered tires, and test with rear altered tires at both the inside and outside positions. However, yaw rate was highest during the tests with a prepared tire placed at the outer rear position. In all tests, the vehicles remained under control of the driver until large steering inputs were made and held.

Documentation of Physical Evidence

Following each run, the physical evidence deposited on the skid pad was documented. During each analyzed test, two or more tire marks were deposited by the test vehicle. In some tests, tire striations were visible. The outer edge of the tire marks was marked with chalk approximately every 20 to 30 feet. The points were then surveyed with a Sokkia Series30R Total station.

Each test was recorded on video and the tire mark evidence was photographed. The outer edge of each tire mark and the rest position was surveyed. In later tests, a drone was used to record the tests from above. Following the tests, photographs of the evidence were captured in grid pattern from the drone.

Analysis of Data

Evidence Diagram

The survey data was used to create an evidence diagram for each test. In later tests, a scaled rectified aerial photograph was compiled from the drone data and used to supplement the survey data. The default cubic spline in Autocad was used to connect the surveyed tire mark points.

Superelevation

Superelevation was considered in each test. The majority of tests were conducted on generally flat surfaces, without superelevation. Two of the test facilities had a minor cross slope which was incorporated into the analysis.

Radius Measurement

Two different radii were considered; the radius of the front outer tire mark and the radius of the center of gravity of the vehicle. The radius of the front outside tire mark was measured on the diagram with a three-point arc (portion of a circle). An arc corresponding to a chord of 50 feet (approximately 15 meters) was used. The middle ordinate was then measured. In a few tests, the middle ordinate was less than 12 inches (0.3 meters), so a longer arc was used until these criteria was met. In some rear disablements, the chord and middle ordinate requirements resulted in a separation of front and rear tire marks (off-tracking) more than one half of the track width. This will be discussed later.

Vehicle positions were reconstructed based on the tire mark evidence in order to determine the path of the center of gravity. The longitudinal center of gravity for each vehicle was calculated from the weight distribution. The lateral center of gravity was approximately 1 inch or less from the centerline of the vehicle. The center of gravity was assumed to be centered on the lateral axis of the vehicle, as is common in practice in accident reconstruction. The scaled vehicle model was then aligned with the tire marks. The longitudinal center of the outside tired edges were aligned as closely as possible with the outside edges of the tire marks. Three vehicle positions approximately 25 feet apart were used. The first position was aligned to the tire marks when the outside marks began to diverge. A three-point arc was then aligned to the center of gravity positions. A chord and middle ordinate were drawn. If a middle ordinate of 12 inches was not achieved, the vehicles were spaced further apart (remaining equidistance) until a middle ordinate of 12 inches was achieved.

Equivalent Vehicle Coefficient of Friction

The roadway friction coefficient was calculated from the skid to stop tests in which all the tires had tread and in tests in which the tread was removed from all the tires. This testing was performed with both vehicles. Figures 9 and 10 depict a sample of the acceleration data from the Expedition and

FIGURE 9 Expedition skid to stop tests.

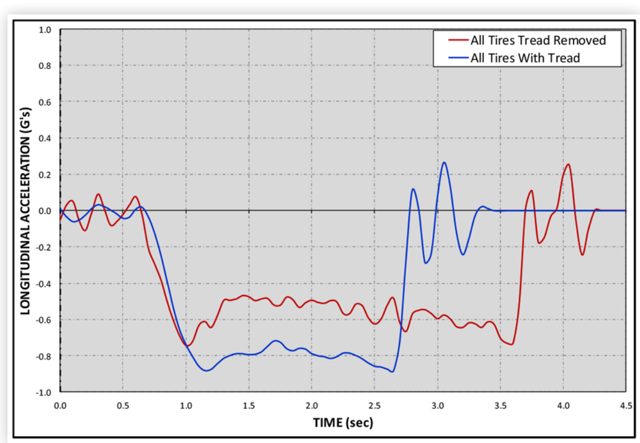
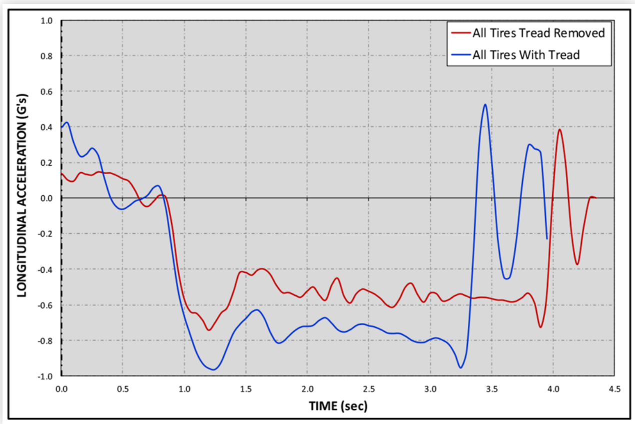


FIGURE 10 Malibu skid to stop tests.



Malibu tests, respectively. The blue line in each plot is from a test with four tires all with tread. The red curve is from a test with tread removed from all tires. The coefficient of friction was computed in accordance with SAE J2505. Table 3 depicts the average friction from each test series. Removing the tread from a tire on the Malibu resulted in a friction reduction to 64% of the friction with tread. For the Expedition, removing the tread resulted in 74% of the friction with tread.

For analysis of the tests with all four tires with tread, the friction coefficient calculated from the skid to stop test with all tires with tread at that test site was used.

For analysis of tests with an altered tire, several different methods for calculating the equivalent vehicle coefficient of friction were considered:

A. The friction coefficient calculated from the skid to stop tests with all tires unaltered was assumed for the vehicle.
B. The friction of the subject tire was reduced (to 64% for Malibu tests, to 74% for Expedition tests) based on the results of the skid to stop testing. The unaltered test specific tire friction was used for the other three tires. The equivalent vehicle friction was then calculated according to the static corner weight distribution of the vehicle.
C. The friction of the subject axle was reduced (to 64% for Malibu tests, to 74% for Expedition tests) based on the results of the skid to stop testing. The unaltered test specific tire friction was used for the tires on the other axle. The equivalent vehicle friction was then calculated according to the static axle weight distribution of the vehicle.
D. The friction coefficient calculated from the skid to stop tests with four tires with tread removed was assumed for the vehicle (to 64% for Malibu tests, to 74% for Expedition tests).

TABLE 3 Friction coefficients of the various tests.

Expedition Friction Testing			
Test Date	**With Tread**	**Tread Removed**	**Tread Removed Percentage**
5/31/2012	0.725	0.536	74%
7/24/2018	0.750	0.554	74%
Malibu Friction Testing			
Test Date	**With Tread**	**Tread Removed**	**% Reduction**
6/24/2011	0.742	na	na
9/28/2018	0.766	0.488	64%

Coasting Adjustment

Brach proposed increasing the calculated speeds by 4.6% if the vehicle was coasting [9]. Each test was analyzed with and without the coasting adjustment.

Analysis Results

Four Unaltered Tires

There were 14 tests with four unaltered tires, 8 tests with the Malibu and 6 tests from the Expedition that were analyzed. In the majority of tests, the rear tires off-tracked from the front tires. In one Expedition test, only one tire mark was visible, so there was no evidence that the rear tires off-tracked from the front tires. Analysis of this test resulted in inaccurate speed estimates (41% error), as expected, since off tracking is a requirement of using the method. For the tests that did off track, a radius measurement from the center of gravity gave the best results. An arc with a chord length of 50 feet was used. In all tests, the middle ordinate was greater than one foot. The off-tracking did not exceed one half track width over the chord distance. The sideslip angles at the end of the analyzed arc ranged between 4 and 24 degrees. The friction from tests with all the tires with tread were used for the analysis. A coasting adjustment, an increase of 4.6% to the calculated results, increased the accuracy of the calculations [9]. Excluding the test that did not off track, analyzed speeds were between -8% and 5% of actual speeds after the coasting adjustment was made. The results are depicted in Figures 11 and 12. Diagrams of the four unaltered tire tests appear in Appendix A.

FIGURE 11 Analysis results - Malibu, four unaltered tires.

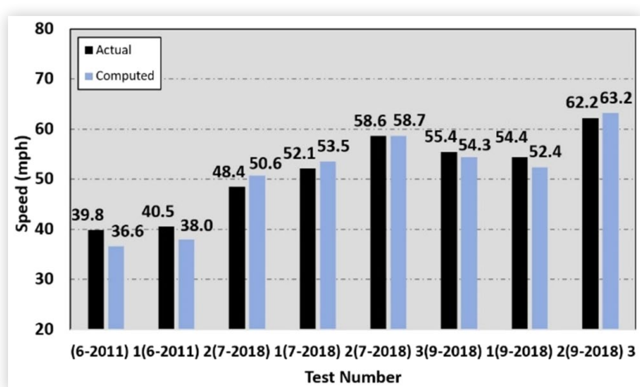
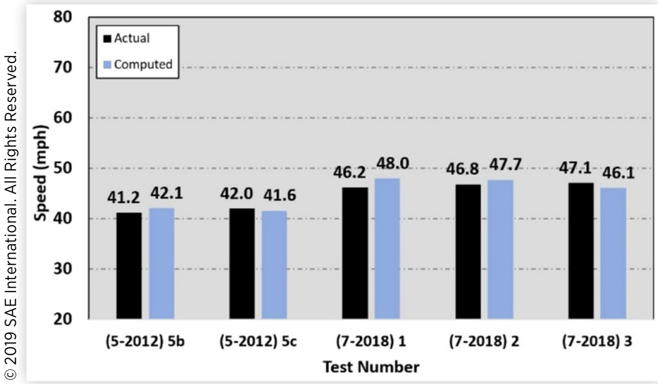


FIGURE 12 Analysis results - Expedition, four unaltered tires.



Front Outside Tire - Tread Removed

Tandy and Dickerson [32,33] found that in steady state cornering, a vehicle experienced a significant understeer effect if a tire without tread was placed in the front outside position. If the tire without tread was located at the inner front tire, the vehicle behaved similarly to a vehicle with four tires with tread. Tandy's and Dickerson's testing confirms that the outside front tire is responsible for the bulk of the steering forces.

A description of steady state cornering is helpful to consider for this configuration. If a driver initiates and holds a steering input at highway speed, the front tires develop slip angle. The vehicle then begins to yaw, resulting in the rear tires developing slip angle. Once the rear tires reach a sufficient slip angle to balance the forces (moments) of the front tires about the center of gravity, steady state is achieved. Thus, the force demanded of the rear tires, are determined by the force achieved by the front tires (and the distance between the tires and the center of gravity). In the case of a front outside tire disablement, the maximum frictional capabilities of the front axle is reduced by the presence of the disabled tire. Therefore, the force levels required by the rear tires to achieve steady state will be reduced as well. Theoretically, the friction capabilities of the front outside tire should limit the cornering of the vehicle. Therefore, a vehicle equivalent friction equal to the friction capabilities of a tire without tread was included in the analysis (friction method D).

There were 9 tests with a front outside altered tire, 3 tests with the Malibu and 6 tests from the Expedition. A radius measurement from the center of gravity gave the best results. In most tests, an arc with a chord slightly greater than 50 feet was required to satisfy a middle ordinate of one foot. The off-tracking did not exceed one half track width over the chord distance. The sideslip angles at the end of the analyzed arc ranged between 1 and 6 degrees. The coefficient of friction of a tire with tread removed applied globally to the vehicle gave the best results for the front tire tests (friction method D), consistent with the theoretical discussion above. No coasting adjustment was needed. In two tests, the rear tires did not track outside the front tires. Analysis of the tests that did not off-track resulted in high errors (33% to 52%). Excluding the tests that did not off track, analyzed speeds were between -8% and 5% of actual speeds, as depicted in Figures 13 and 14. Diagrams for all front outside tire tests appear in Appendix B.

In analyzing the tests, the friction reduction was tire (vehicle) specific. Taking tread off the tires of the Expedition resulted in a friction reduction to 74% of the unaltered tires. The friction for the Malibu tires was reduced to 64% of the unaltered tire friction. These reductions were used for the Expedition tests and Malibu tests, respectively. Using the actual friction reduction allowed for a comparison of the different methods presented, and it was shown that the critical speed method could be extended to the case with a front tire without tread. However, in practice it is unlikely that the actual reduction in friction would be known. The tests were reanalyzed using the most accurate method - three-point radius of the center of gravity path, reduced friction used for the entire vehicle, no coasting adjustment - using the entire range for the friction reduction for all tests (64% to 74% of unaltered tire friction). Ranging the friction reduction increased the error for the analysis, to between -8% and 13%. In practice, the entire range of friction reductions (64% to 74% of unaltered tire friction) should be used unless tire specific testing is conducted.

FIGURE 13 Analysis results - Malibu, front outside tire tread removed.

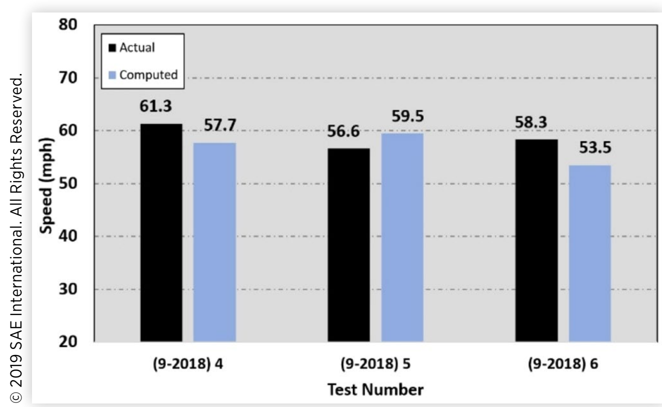
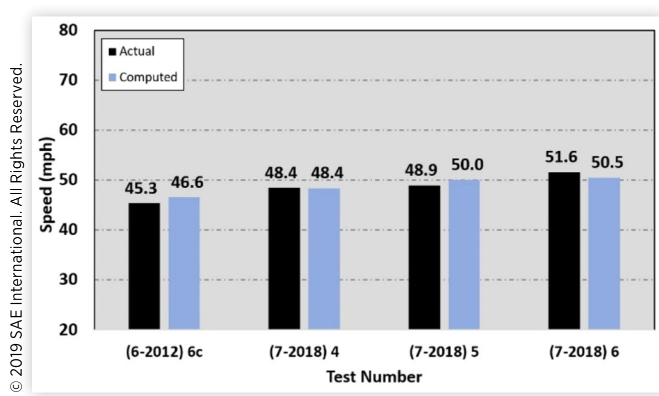


FIGURE 14 Analysis results - Expedition, front outside tire tread removed.



Rear Outside Tire - Tread Removed

Each vehicle with an altered tire in the outside rear position spun out during the testing. The rear tires off-tracked from the front tires in all cases, demonstrably more than in the front tire testing and testing with four unaltered tires. There was also a larger difference between the radial path of the outside front tire and the radial path of the vehicle center of gravity in the outer rear tire tests. It was assumed that reducing the vehicle equivalent friction by some amount would be necessary since the vehicles cornering capacities were reduced. The best combination of radius and equivalent friction coefficient was determined empirically.

It was found that using the radius of the vehicle's center of gravity gave the best results. An arc with a chord length of 50 feet was used to determine the center of gravity path radius. In all tests, the middle ordinate was greater than one foot. The off-tracking exceeded one half track width over the chord distance in most tests but was approximately one whole track width or less in all tests.[2] At the end of the measured arc, the vehicle sideslip angle ranged between 10 and 35 degrees. For the equivalent vehicle friction, reducing the friction of the subject tire only based on the static corner weight of the vehicle worked best (friction method B). A coasting adjustment, an increase of 4.6% to the calculated results, increased the accuracy of the calculations [9].

There were 13 tests with a rear outside altered tire, 6 tests with the Malibu and 7 tests from the Expedition. Analyzed speeds were between -10% and 9% of actual speeds after the coasting adjustment was made, as depicted in Figures 15 and 16. Diagrams for all rear outside tests appear in Appendix C.

The friction reduction was tire (vehicle) specific. Taking tread off the tires of the Expedition resulted in a friction reduction to 74% of the unaltered tires. The friction for the Malibu tires was reduced to 64% of the unaltered tire friction. These reductions were used for the Expedition tests and Malibu tests, respectively. However, in practice it is unlikely that the actual reduction in friction would be known. The tests were reanalyzed using the most accurate method - three point radius of the center of gravity path, reduced friction used for the subject tire, a coasting adjustment was used - using the entire range for the friction reduction for all tests (64% to 74% of unaltered tire friction). Ranging the friction reduction did not significantly change the error (still -10% to 9%). It is recommended that the average friction reduction, to 69% of the friction of an unaltered tire, should be used in practice. For the rear outside tire configuration, the analysis was relatively insensitive to the friction reduction.

FIGURE 15 Analysis results - Malibu, rear outside tire tread removed.

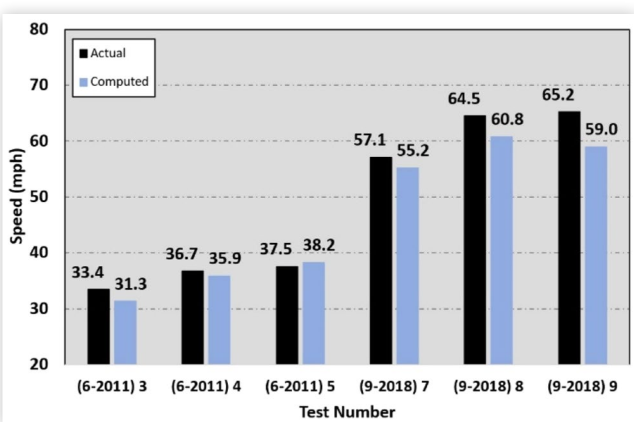
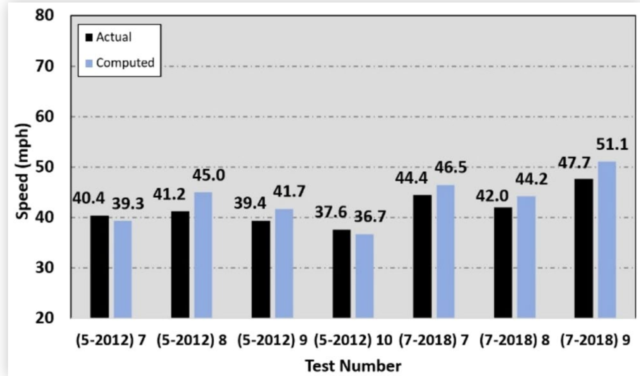


FIGURE 16 Analysis results - Expedition, rear outside tire tread removed.



Rear Inside Tire - Tread Removed

Tandy and Dickerson found minimal handling differences in steady state testing when a tire without tread was placed at an inside rear position [32,33].

There were 5 tests with a rear inside altered tire, all with the Malibu. The rear tires off tracked from the front tires in all cases. The radius of the center of gravity gave the most accurate results. In some tests, an arc with a chord slightly greater than 50 feet was required to satisfy a middle ordinate of one foot. The off-tracking did not exceed one track width over the chord distance.[3] The range of sideslip angles at the end of the analyzed arc was 8 to 25 degrees. Using the friction coefficient calculated from the skid to stop tests with all tires unaltered applied to the entire vehicle gave the best results, consistent with the work of Tandy and Dickerson [32,33]. A coasting adjustment, an increase of 4.6%, increased the accuracy of the calculations. The analyzed speeds were between -5% and 3% of actual speeds after the coasting adjustment was made, as depicted in Figures 17. Diagrams for all rear inside tests appear in Appendix D.

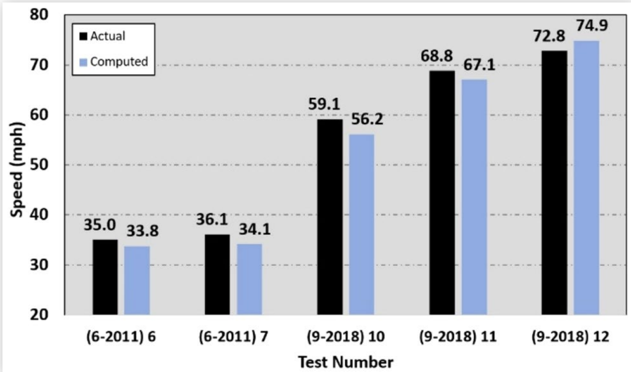
Analysis Discussion

A friction reduction was required for the front outside tire tread removed and rear outside tread removed configurations. If the actual friction reductions were used (to 64% for the

[2] 1.1 x track width was the largest separation in this test series.

[3] 0.75 x track width was the largest in this test series.

FIGURE 17 Analysis results - Malibu, rear inside tire tread removed.

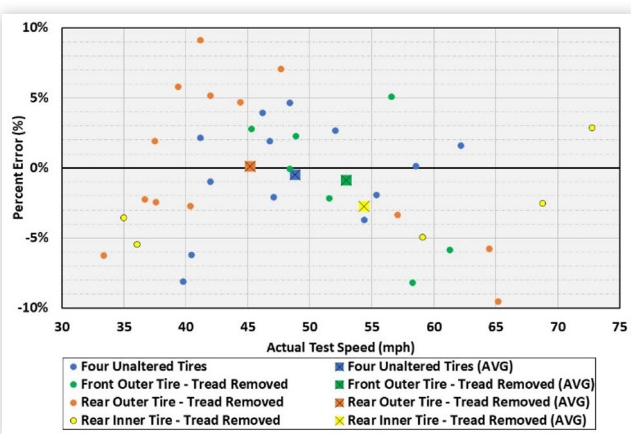


Malibu and to 74 % for the Expedition), the error range of all four test configurations was -10% to 9%, within the accepted -13.5% to 10% range assumed for a vehicle with four unaltered tires [1]. Figure 18 depicts the analysis of all of the tests. A different color is used for each variation of tire condition. The actual test speed is plotted on the x-axis. Percent error of that test is plotted on the y-axis. The error percentage of the Critical Speed Formula was not test speed dependent. In general, the Critical Speed Formula tended to underestimate the actual speed or bracket the correct speed.

For the front outside tire tread removed configuration, ranging the friction reduction (to 64% to 74% of unaltered tire friction) increased the error range from -8% to 5%, to -8% to 13%. Since the friction reduction applied to the entire vehicle for the front outside tests, the results were sensitive to the reduction. Therefore, the entire range should be used in practice.

On the other hand, the rear outside tests were insensitive to the friction reduction. In practice, using the average friction reduction (to 69%) would be adequate. To highlight the insensitivity of the friction reduction in the rear outside tire tread removed configuration, the friction of the tire was reduced to 20% of full tread friction, far less than the actual reduction (64% to 74%). Reducing the friction to 20% of actual at the subject tire resulted in a range of errors of -13% to 1%, still within the accepted critical speed analysis range (-13.5% to 10%).

FIGURE 18 Critical Speed Formula Percent Error vs Test Speed - All Tests.



For the rear outer tire tread removed cases, adjusting the friction of the subject tire alone gave the best results (CG path radius, coasting adjustment). Adjusting the friction of the rear axle also gave acceptable results (-13% to 5% error), albeit more conservative. In some real world cases, larger lateral weight shift may result in the adjustment to the entire rear axle being more representative.

In both the four unaltered tire tests and the prepared front outer tire tests, off tracking was not always achieved. A lack of off-tracking was an indication that maximum lateral acceleration was not achieved. When these tests were analyzed using the guidelines set forth above, the Critical Speed Formula overestimated the vehicle speed by as much as 52%. The Critical Speed Formula should not be used unless evidence of off-tracking exists (separation of the rear tire mark outside the front tire mark).

Appendix E depicts the square root of the sum of the squares error and ranges of error for all tests (both vehicles) in each test configuration for each analysis method. The results of Appendix E were used to determine the preferable method for each test configuration, which is highlighted in blue. For the unaltered tire configuration, front outside tire tread removed configuration and rear inside tire tread removed configuration, a single method gave the most accurate results (highlighted in blue). For the rear outside tire tread removed configuration, two methods resulted in the same square root of the sum of squares error. A second criterion, the method which better centered the analysis results around the actual speeds, was used to determine the preferred method in the rear outside tire tread removed configuration. Analysis results of all tests appear in Appendix F. The cells highlighted in blue were the recommended method. The recommended method was selected considering the results from all test of both vehicles. Therefore, the recommended procedure doesn't necessarily give the most accurate results for any individual test. In Appendix F, tests in red font did not achieve off-tracking and were not included in the results in the body of this paper.

For all test conditions, the center of gravity path yielded the best results. This not surprising since the critical speed method treats the vehicle as a particle. The path of the center of gravity is most representative of the entire vehicle and is the point of the vehicle least affected by developing sideslip angle. Other authors have made adjustments to the radius of the front tire mark, subtracting ½ track width for example, in an attempt to better approximate the path of the center of gravity. However, as sideslip angle develops, the discrepancy between the radius of the front outer tire mark and the radius of the center of gravity increases, and the difference cannot be captured by a constant adjustment. This is especially relevant in cases where tread was removed from the rear outer tire. Even in cases where the sideslip angle reached 35 degrees (a separation in leading tire marks of approximately 1.1 times the track width), the critical speed formula still yielded speed estimates within a satisfactory error range. Previous authors have proposed in order to use the critical speed formula, over the segment of measured chord, the separation of the tire marks should be less than half the track width of the vehicle. It was found that this recommendation could be relaxed when using the path of the center of gravity.

The testing and analysis conducted above shows that the critical speed method can be extended to cases of a tire without tread at various tire locations. Adjustments had to be made to account for the lateral friction capabilities of the vehicle in each configuration. Four different adjustments were considered to determine which method was most accurate for that configuration;

A. Full tire friction for all tires.
B. Subject tire friction reduced.
C. Subject axle friction reduced.
D. Friction reduced for all tires.

In each test, the tire remained inflated. Although other tire disablement types (airloss for example) were not tested here, any type of disablement will reduce the lateral capabilities of the vehicle to some degree. It is likely that the critical speed formula could be extended to other types of disablements as well, so long as a reasonable equivalent lateral friction could be determined. However, additional work is required to extend the critical speed formula to other disablement types.

The coasting adjustment proposed by Brach, increasing the analyzed results by 4.6 percent, increased the accuracy of all the test conditions except the tests with tread removed from a front outer tire. These front outer tire tread removed tests also had the least amount of vehicle sideslip, with sideslip angles between 1 and 6 degrees. Because small sideslip angles were generated, the vehicles in these tests decelerated minimally during the analyzed arc distance. By comparison, vehicles in all other test configurations developed more sideslip, and decelerated more. Although not analyzed specifically, these authors suspect that minimal speed loss during the analyzed arc distance was the reason that a coasting adjustment was not needed. No braking or accelerating tests were conducted during this testing. It is unknown if the braking or acceleration adjustment recommended by Brach would apply in the case of a yaw with tire tread detachment.

This paper explored speed analysis of yawing vehicles. The authors were not specifically examining the controllability of the vehicles in these test series, however, a few findings are worth mention. In the 2018 tests with the Malibu, higher test speeds were achieved by starting on the outer test track and navigating a left curve onto the test pad surface. During this left curve, lateral accelerations over 0.6 g's were achieved with a rear outside tire with tread removed. The vehicle was kept under the driver's control during this turn. The vehicles were kept under the driver's control until a larger steering input was made and held. Controllability during tread detachments was studied for both these vehicles more extensively in previous publications [34,35].

Recommendations

Regardless of the tire condition, the following first four steps should be followed:

1. There should be at least two curved tire marks visible, they should be from the outside wheels, and there should be evidence that the marks are yaw marks (not skid marks). This evidence can be lateral striations or scratches, or the circumstances of the crash. There should be clear evidence that the rear tires were tracking outside the front tires.
2. Three vehicle positions should be reconstructed based on the tire mark evidence, equally spaced with center of gravity positions approximately 25 feet apart. The first vehicle should be positioned at the earliest point corresponding to Guideline 1. The longitudinal center of the outside edges of the tires should be aligned as closely as possible with the outside edges of the tire marks.
3. The radius of the path of the center of gravity should be measured by drawing a three-point arc through each center of gravity.
4. Draw a chord (line between the beginning and end of the arc) and measure the middle ordinate. If the middle ordinate is less than 0.3 m (1 ft), evenly increase the spacing between the vehicle models and retrace the vehicle center of gravity path with a new arc until the middle ordinate is 0.3 m (1 ft).

Steps 5 and beyond are situationally dependent:

Four Unaltered Tires

5. The equivalent vehicle friction should be calculated from a skid to stop test at the scene in accordance with J2505 with all tires with tread. If skid to stop data is not available, a range of values should be used based on available literature.
6. If the vehicle is coasting, the calculated speed should be adjusted +4.6%.
7. This test series indicates an uncertainty range of -8% to 5% when these guidelines are observed. The Critical Speed Task Force who authored J2969 determined that the method is accurate within -13.5% (underestimate in speed) to +10% (overestimate in speed) when used correctly. J2969 does note that if the vehicle is coasting, an adjustment can be made to reduce the level of uncertainty.

Front Outside Tire - Tread Removed

5. The equivalent vehicle friction should be calculated by assuming the tread removed friction for the entire vehicle. Reduce the equivalent vehicle friction to 64% to 74% of the friction of a tire with tread, or a specific percentage determined through testing. The friction of tires with tread should be determined from a skid to stop test at the scene in accordance with J2505 with all tires with tread. If skid to stop data is not available, a range of values should be used based on available literature.
6. No coasting adjustment should be made.
7. By incorporating the full range of friction reduction (64% to 74%of friction of unaltered tires), an error range of -8% to 13% was achieved.

Rear Outside Tire - Tread Removed

5. The equivalent vehicle friction should be calculated by reducing the friction of the subject tire to compensate for the tread detachment. Reduce the friction of the outside rear tire to 69% of the friction of the tires with tread. Calculate the equivalent vehicle friction based on the static weight distribution of the vehicle. The friction of tires with tread should be determined from a skid to stop test at the scene in accordance with J2505 with all tires with tread. If skid to stop data is not available, a range of values should be used based on available literature.
6. If the vehicle is coasting, the calculated speed should be adjusted +4.6%.
7. By following these guidelines, a uncertainty range of approximately -10% to 9% was achieved.

Rear Inside Tire - Tread Removed

5. The equivalent vehicle friction should be calculated from a skid to stop test at the scene in accordance with J2505 with all tires with tread. If skid to stop data is not available, a range of values should be used based on available literature.
6. If the vehicle is coasting, the calculated speed should be adjusted +4.6%.
7. By following these guidelines, an uncertainty range of approximately -5% to 3% was achieved.

Recommendation Summary

Table 4 depicts a summary of the guidelines for each method. For all categories except the front outside tire tread removed, increasing the calculated speed using a coasting adjustment multiplier of 4.6% increased the accuracy of the results. Not correcting with a coasting adjustment would make the results more conservative (underestimate) compared to the actual speed.

TABLE 4 A summary of recommendations for using the Critical Speed Formula for a vehicle with a tire without tread at various tire positions.

Disablement Position	Radius Measurement Location	Vehicle Equivalent Friction	+4.6% Coasting Adjustment?
None	Center of Gravity	Full Tire Friction (J2505 or Tables)	Yes
Front Outside	Center of Gravity	All Tires Adjusted For Missing Tread (64% to 74% of Friction with Tread)	No
Rear Outside	Center of Gravity	Rear Outside Tire Adjusted For Missing Tread (69% of Friction with Tread)	Yes
Rear Inside	Center of Gravity	Full Friction (J2505 or Tables)	Yes

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Conclusions

1. The Critical Speed Formula was shown to give results within the previously published accepted range of accuracy (-13.5% to 10%) for the cases of four unaltered tires, a rear outside tire with tread removed, and a rear inside tire with tread removed, provided that case specific guidelines are observed.
2. For the case of a front outer tire with tread removed, the results were shown to be relatively sensitive to the friction reduction. The critical speed formula gave results with a range of error of -8% to 13%.
3. For the case of a rear outside tire with tread removed, the results were relatively insensitive to the amount of friction reduction of the subject tire. A reduction in friction for subject tire to 69% of full tread friction was sufficient.
4. For the case of an inside rear tire with tread removed, assuming full friction, as if no disabled tire was on the vehicle, gave the best results.
5. The center of gravity radius, measured with a three-point arc, gave the most accurate results in all test configurations.
6. The critical speed formula does not need to be restricted to only cases where the separation of the tire marks is less than half the track width of the vehicle over the measured chord length, provided that the center of gravity path radius is used. Using the center of gravity path yielded reasonable accurate results even when the sideslip angle reached 35 degrees (1.1 x track width of separation) over the chord.
7. Acceptably accurate speed calculations are achievable when using center of gravity radii measured with a three-point arc from a scaled diagram.
8. In several tests, two large steering inputs were made, and the critical yaw was the result of the second steering input. The Critical Speed Formula gave acceptably accurate results in the case of multiple steering inputs.

References

1. SAE Data Collection and Archiving Standards Committee, "Use of the Critical Speed Formula," SAE Recommended Practice J2969, Rev Jan. 2017.
2. Amirault, G. and MacInnis, S., "Variability of Yaw Calculations from Field Testing," SAE Technical Paper 2009-01-0103, 2009, doi:10.4271/2009-01-0103.

3. Asay, A. and Woolley, R., "Rollover Testing of Sport Utility Vehicles (SUVs) on an Actual Highway," SAE Technical Paper 2010-01-0521, 2010, doi:10.4271/2010-01-0521.
4. Bartlett, W., Wright, W., Masory, O., Brach, R. et al., "Evaluating the Uncertainty in Various Measurement Tasks Common to Accident Reconstruction," SAE Technical Paper 2002-01-0546, 2002, doi:10.4271/2002-01-0546.
5. Bartlett, W. and Wright, W., "Summary of 56 Recent Critical Speed Yaw Analysis Tests, Including ABS and Electronic Stability Control on Pavement, Gravel, and Grass: Corrected," *Accident Reconstruction Journal* 29-32, 2008.
6. Bellion, P., "Project Y.A.M. (Yaw Analysis Methodology) Vehicle Testing and Findings - Victoria Police, Accident Investigation Section," SAE Technical Paper 970955, 1997, doi:10.4271/970955.
7. Bohan, T. and Keierleber, J., "Speed from "Skuffs" on a Grassy Field: A Case for Dr. Daubert?" in *Proceeding of the American Academy of Forensic Sciences, Annual Meeting*, Atlanta, GA, Feb. 11-16, 2002.
8. Brach, R., "An Analytical Assessment of the Critical Speed Formula," SAE Technical Paper 970957, 1997, doi:10.4271/970957.
9. Brach, R.M. and Brach, M., *Vehicle Accident Analysis and Reconstruction Methods* (Warrendale: SAE International, 2005), 51-64. ISBN:978-0-7680-0776-3.
10. Cannon, J., "A Study of Errors in Yaw-Based Speed Estimates Due to Effective Braking," SAE Technical Paper 2003-01-0888, 2003, doi:10.4271/2003-01-0888.
11. Cliff, W., Lawrence, J., Heinrichs, B., and Fricker, T., "Yaw Testing of an Instrumented Vehicle with and without Braking," SAE Technical Paper 2004-01-1187, 2004, doi:10.4271/2004-01-1187.
12. Daily, J., Shigemura, N., and Daily, J., *Fundamentals of Traffic Crash Reconstruction Volume 2 of Traffic Crash Reconstruction Series* (University of North Florida, Institute of Police Technology and Management, 2006), 435-470. ISBN:978-1-884566-63-9.
13. Dickerson, C., Arndt, M., Arndt, S., and Mowry, G., "Evaluation of Vehicle Velocity Predictions Using the Critical Speed Formula," SAE Technical Paper 950137, 1995, doi:10.4271/950137.
14. Fischer, W., "Challenging the Critical Speed Formula in Light of the Daubert Decision," SAE Technical Paper 2005-01-3141, 2005, doi:10.4271/2005-01-3141.
15. Fricke, L., *Traffic Crash Reconstruction* Second Edition (Evanston: Northwestern University Center for Public Safety, 2010), 322-386. ISBN:0-912642-03-3.
16. Hague, D., Lambourn, R., and Turner D. "Critical Speed Studies I: The Accuracy of Speeds Calculated Form Critical Curve Marks, and their Striations," in *The Institute of Traffic Accidents Investigation, Proceeding of the 3rd National Conference*, 89-99, Telford, Nov. 14-16, 1997.
17. Hague, D., Turner, D., and Williams, A., "Critical Speed Studies II: The Generation of Tyre Marks by Cornering Vehicles" in *The Institute of Traffic Accidents Investigation, Proceeding of the 3rd National Conference*, 100-110, Telford, Nov. 14-16, 1997.
18. Lambourn, R., "The Calculation of Motor Car Speeds Form Curved Tyre Marks," *Journal of Forensic Science Society* 29(6):371-386, 1989, doi:10.1016/S0015-7368(89)73284-9.
19. Lambourn, R., "Braking and Cornering Effects with and without Anti-Lock Brakes," SAE Technical Paper 940723, 1994, doi:10.4271/940723.
20. Manning, L. and Bentson, L., "Highway Speed Vs. Sideslip (Critical Speed in a Curve)," *Journal of National Academy of Forensic Engineers* 1(2), 1984.
21. Masory, O., Delmas, S., Wright, B., and Bartlett, W., "Validation of the Circular Trajectory Assumption in Critical Speed," SAE Technical Paper 2005-01-1189, 2005, doi:10.4271/2005-01-1189.
22. Reveley, M., Brown, D., and Guenther, D., "A Comparison Study of Skid and Yaw Marks," SAE Technical Paper 890635, 1989, doi:10.4271/890635.
23. Sledge, N. and Marshek, K., "Vehicle Critical Speed Formula - Values for the Coefficient of Friction - a Review," SAE Technical Paper 971148, 1997, doi:10.4271/971148.
24. Yamazaki, S. and Akasaka, T., "Buckling Behavior in Contact Area of Radial Tire Structure and Skid Marks Left by Tires," *JSAE Review* 9(3):51-55, 1988.
25. Yamazaki, S. "Friction Coefficient of Tires and Skid Marks Left by Tires," in *Proceedings of the International Workshop on Traffic Accident Reconstruction, Organized by National Research Institute of Police Science*, Japan, Nov. 12-13, 1998.
26. SAE Data Collection and Archiving Standards Committee "Measurement of Vehicle-Roadway Frictional Drag," SAE Recommended Practice J2505, Rev Aug, 2010.
27. Beauchamp, G., Hessel, D., Rose, N., Fenton, S. et al., "Determining Vehicle Steering and Braking from Yaw Mark Striations," *SAE Int. J. Passeng. Cars - Mech. Syst.* 2(1):291-307, 2009, doi:10.4271/2009-01-0092.
28. Beauchamp, G., Thornton, D., Bortles, W., and Rose, N., "Tire Mark Striations: Sensitivity and Uncertainty Analysis," *SAE Int. J. Trans. Safety* 4(1):121-127, 2016, doi:10.4271/2016-01-1468.
29. Beauchamp, G., Pentecost, D., Koch, D., and Rose, N., "The Relationship between Tire Mark Striations and Tire Forces," *SAE Int. J. Trans. Safety* 4(1):134-150, 2016, doi:10.4271/2016-01-1479.
30. Tandy, D., Ault, B., and Pascarella, R., "Steering and Handling Performance Following a Full Tire Tread Belt Separation," SAE Technical Paper 2012-01-0257, 2012, doi:10.4271/2012-01-0257.
31. Dickerson, C., Arndt, M., and Arndt, S., "Vehicle Handling with Tire Tread Separation," SAE Technical Paper 1999-01-0120, 1999, doi:10.4271/1999-01-0120.
32. Beauchamp, G., Koch, D., and Thornton, D., "A Comparison of 25 High Speed Tire Disablements Involving Full and

Partial Tread Separations," *SAE Int. J. Trans. Safety* 1(2):364-385, 2013, doi:10.4271/2013-01-0776.

33. Koch, D., Beauchamp, G., and Pentecost, D., "Deceleration Rates of Vehicles with Disabled Tires," SAE Technical Paper 2017-01-1427, 2017, doi:10.4271/2017-01-1427.

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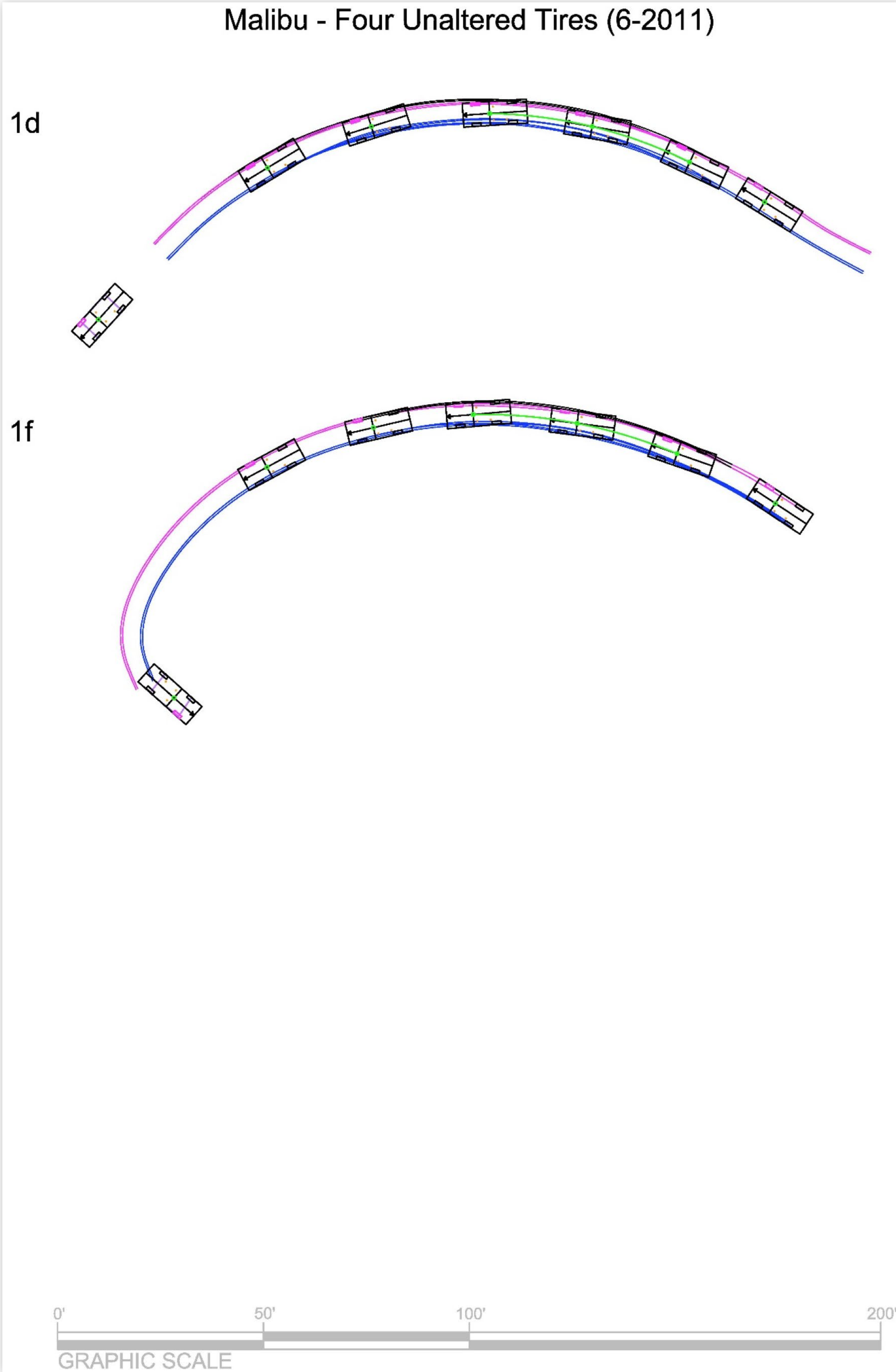
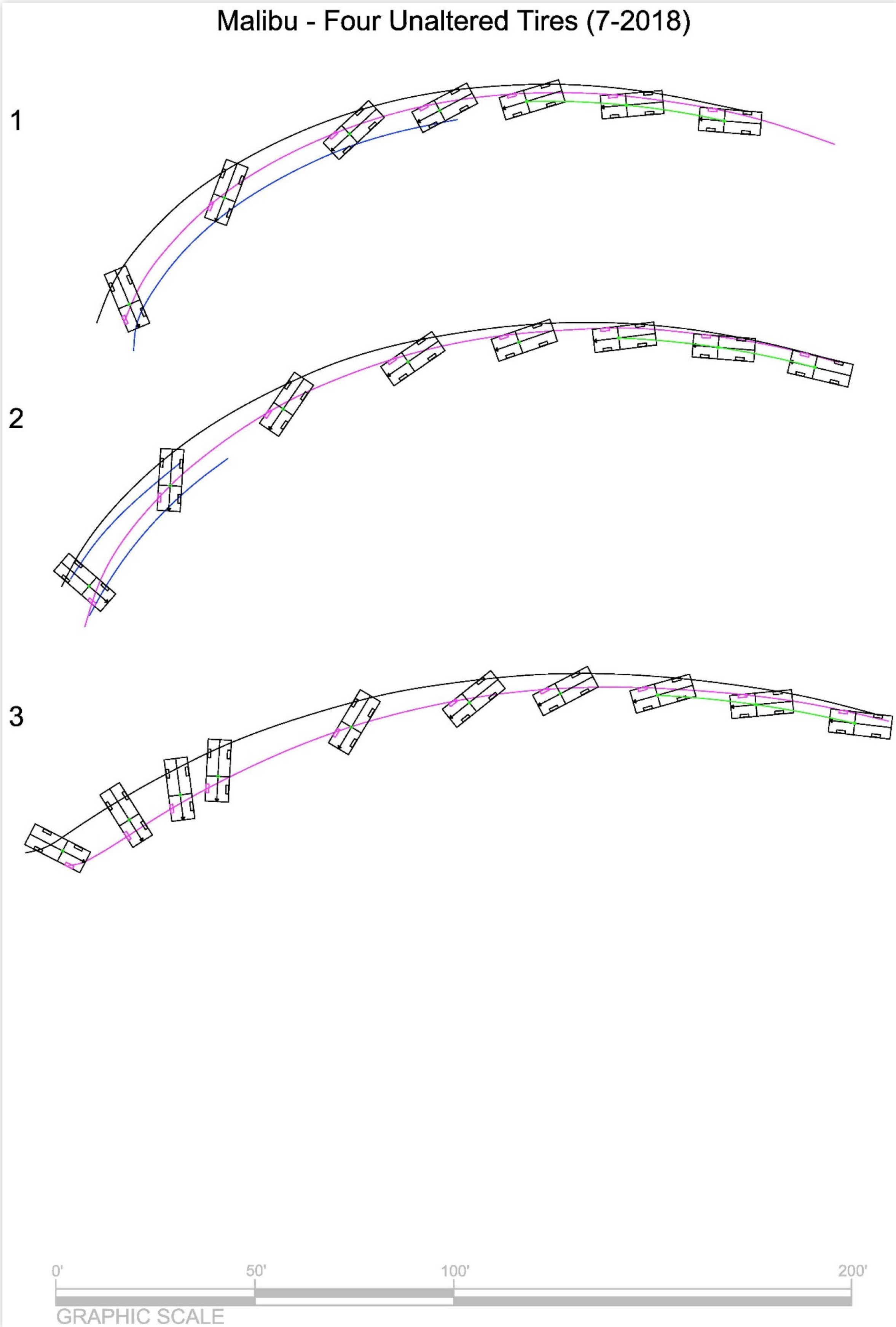
Appendix A


Malibu - Four Unaltered Tires (6-2011)
1d
1f
0'
50'
100'
200'
GRAPHIC SCALE



Malibu - Four Unaltered Tires (7-2018)
1
2
3
0'
50'
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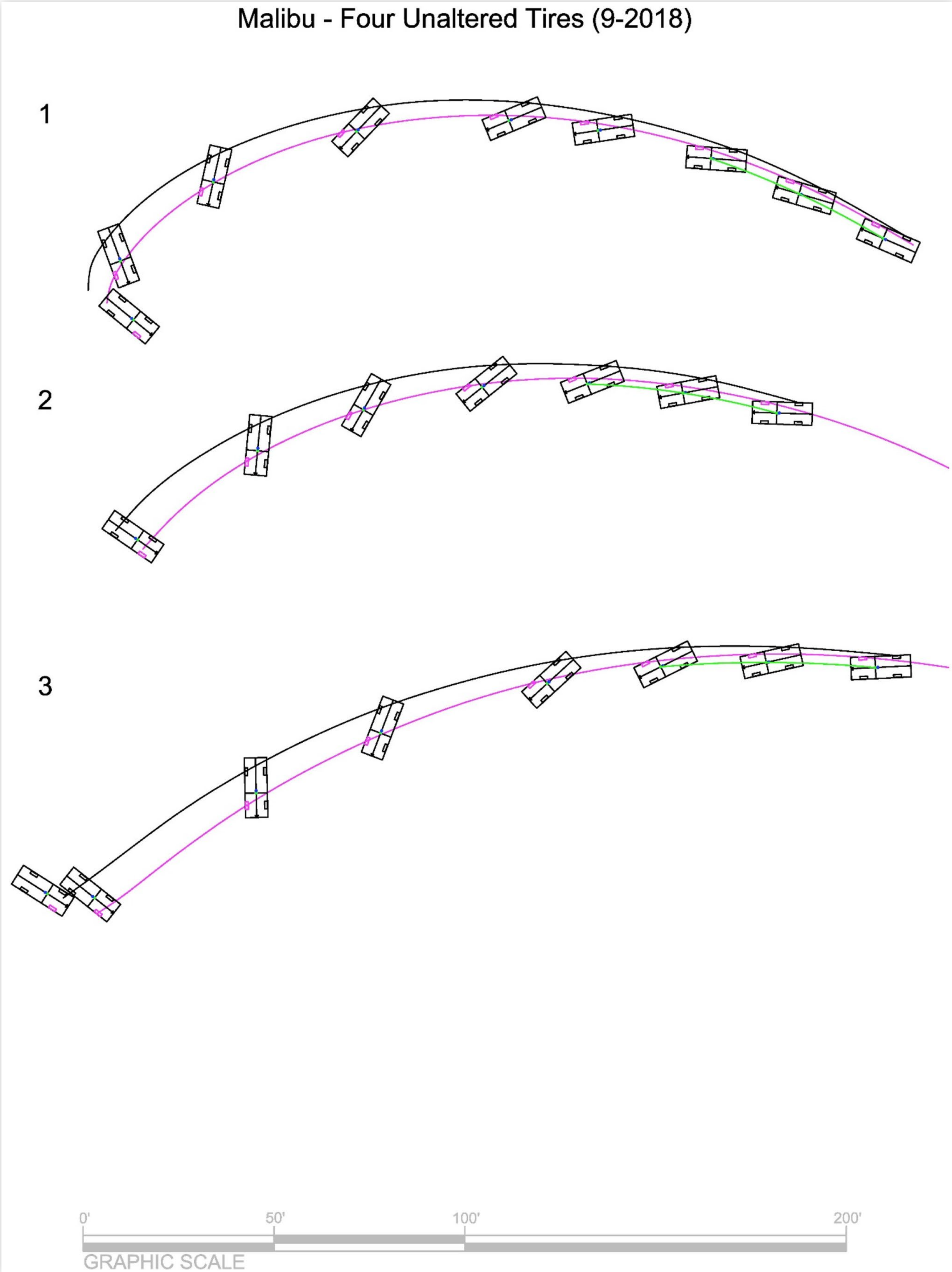
GRAPHIC SCALE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2
3
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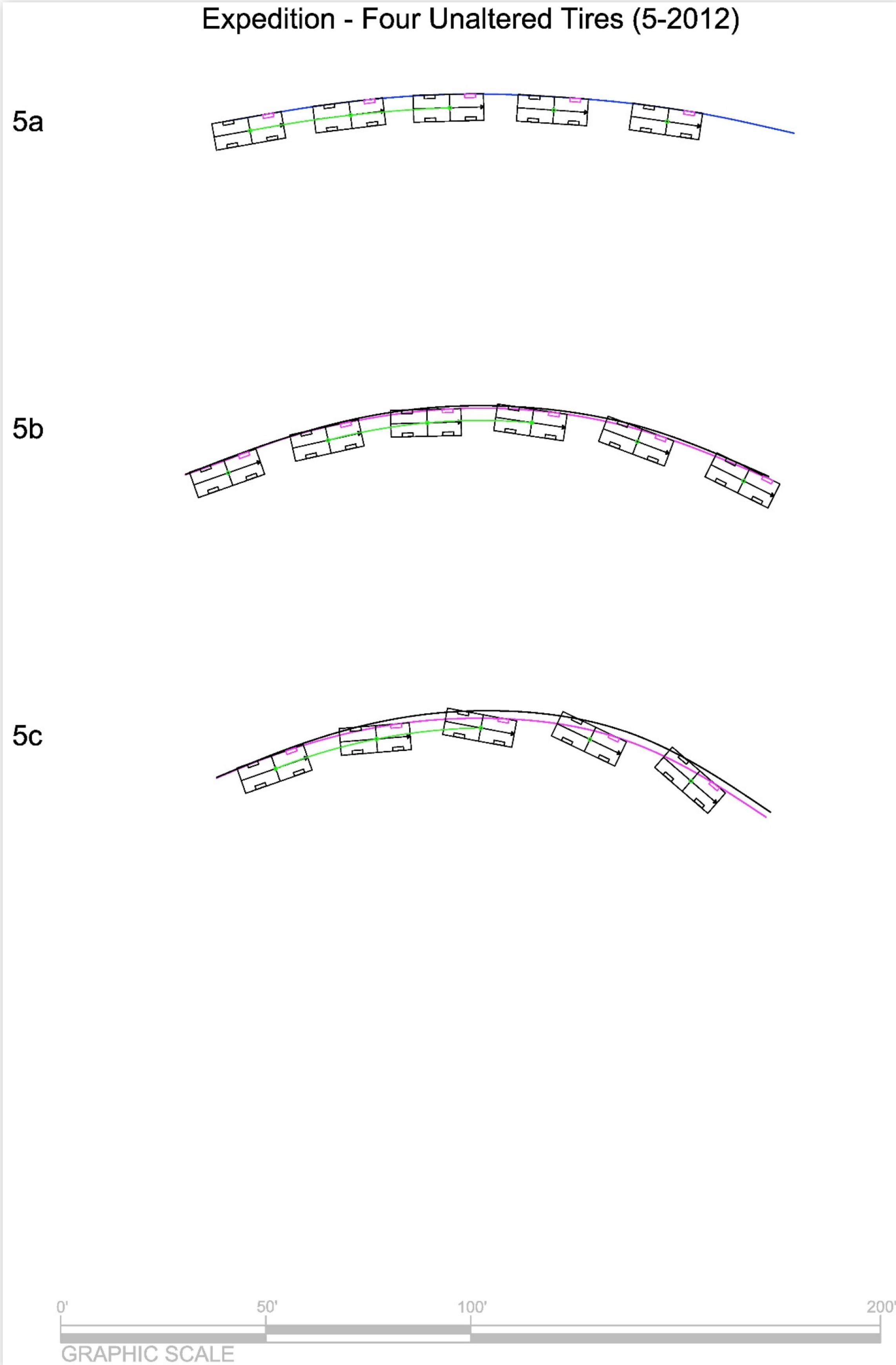
200'
GRAPHIC SCALE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5b
5c
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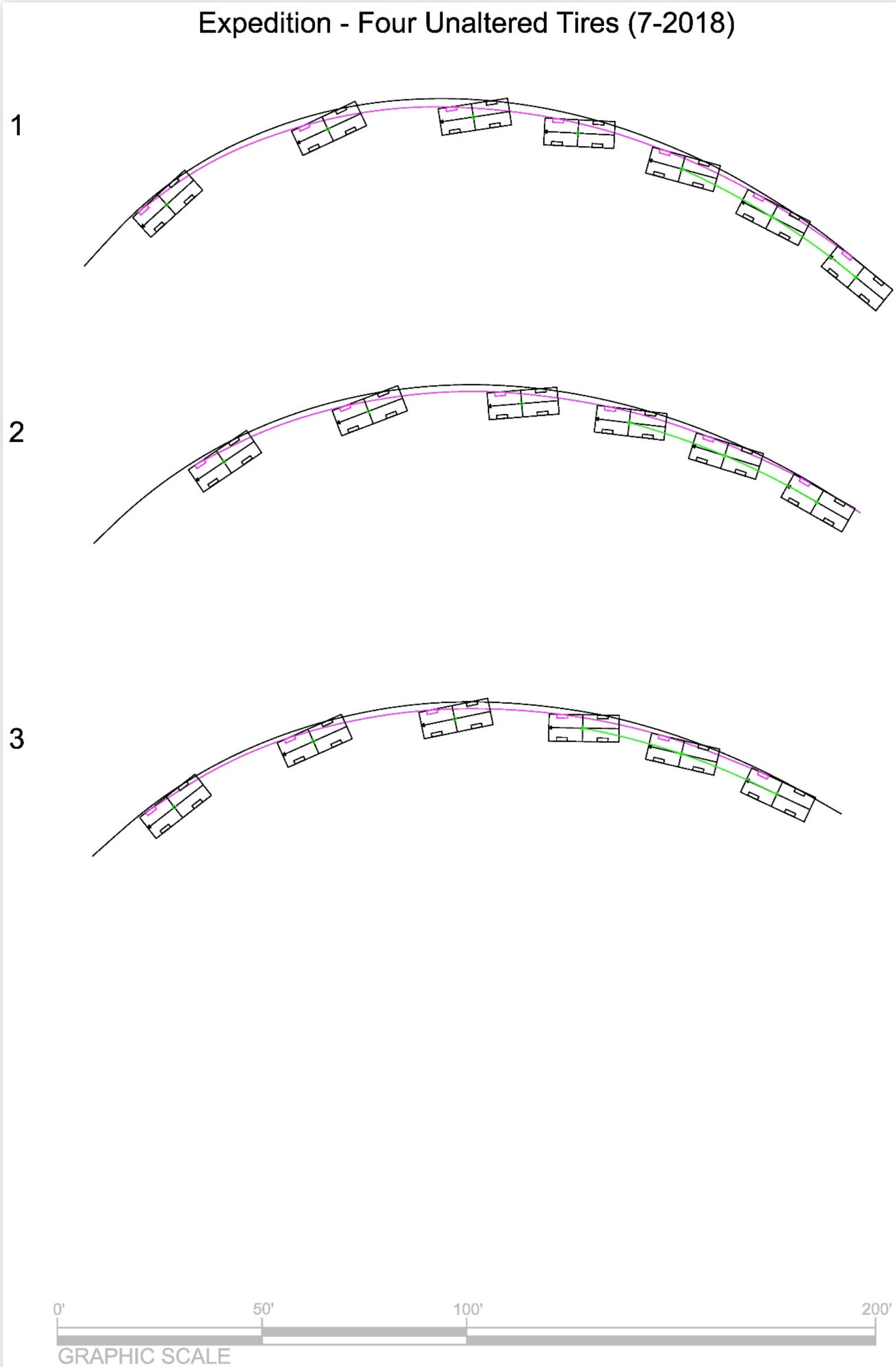
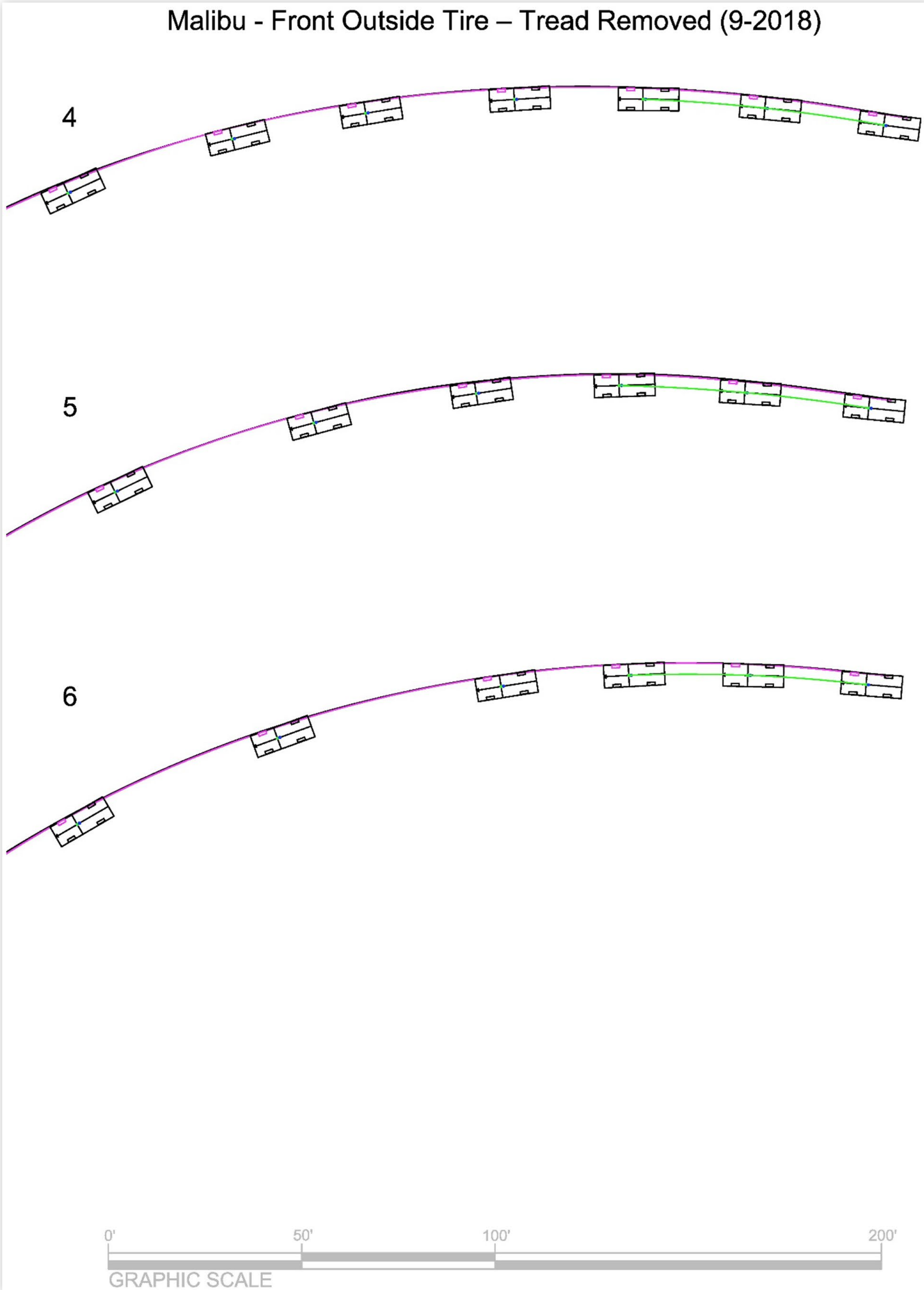
50'
100'
200'
GRAPHIC SCALE


Expedition - Four Unaltered Tires (7-2018)
1
2
3
0'
50'
100'
200'
GRAPHIC SCALE

Appendix B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5
6
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50'
100'
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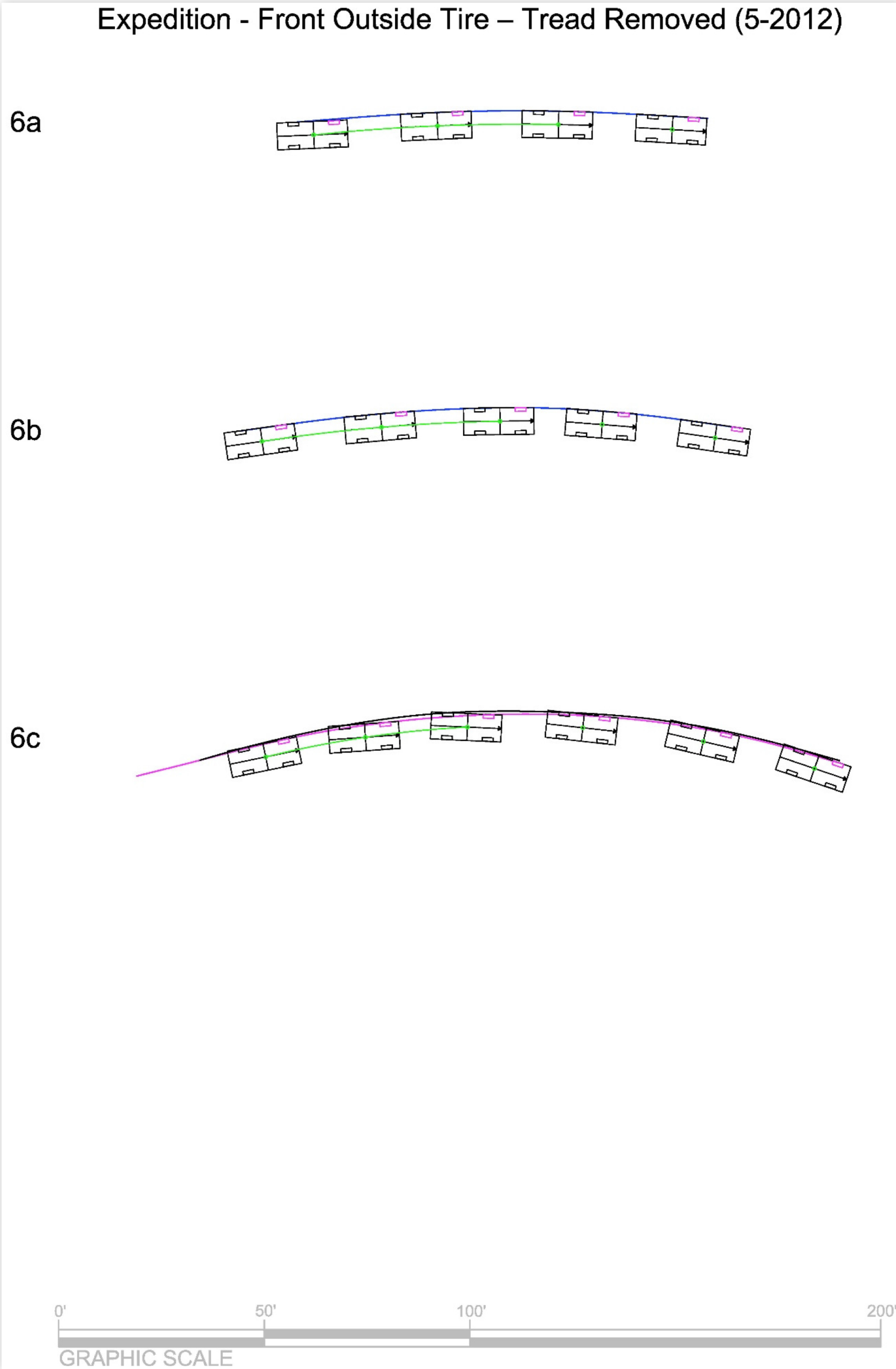
GRAPHIC SCALE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6b
6c
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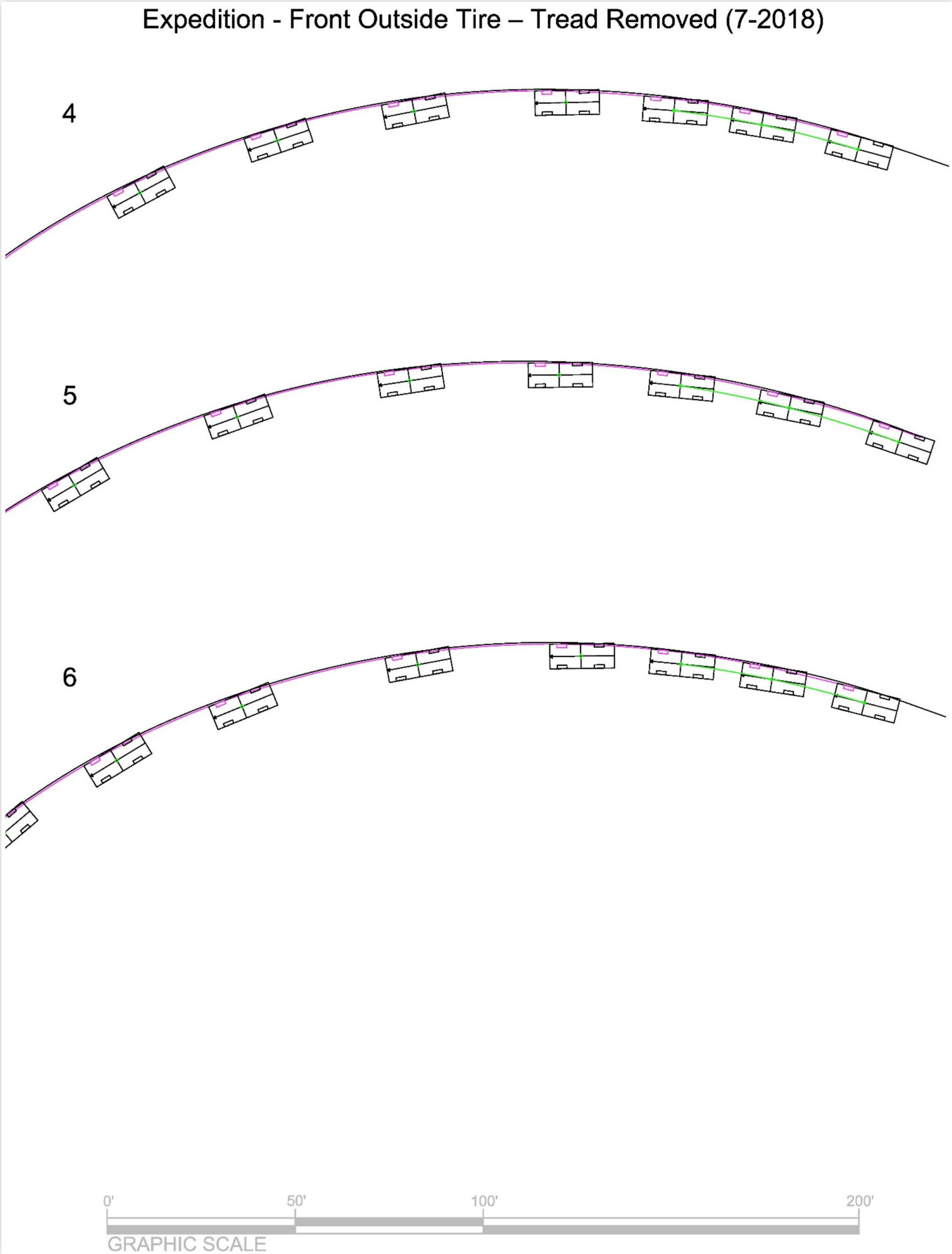
GRAPHIC SCALE


Expedition - Front Outside Tire – Tread Removed (7-2018)
4
5
6
0'
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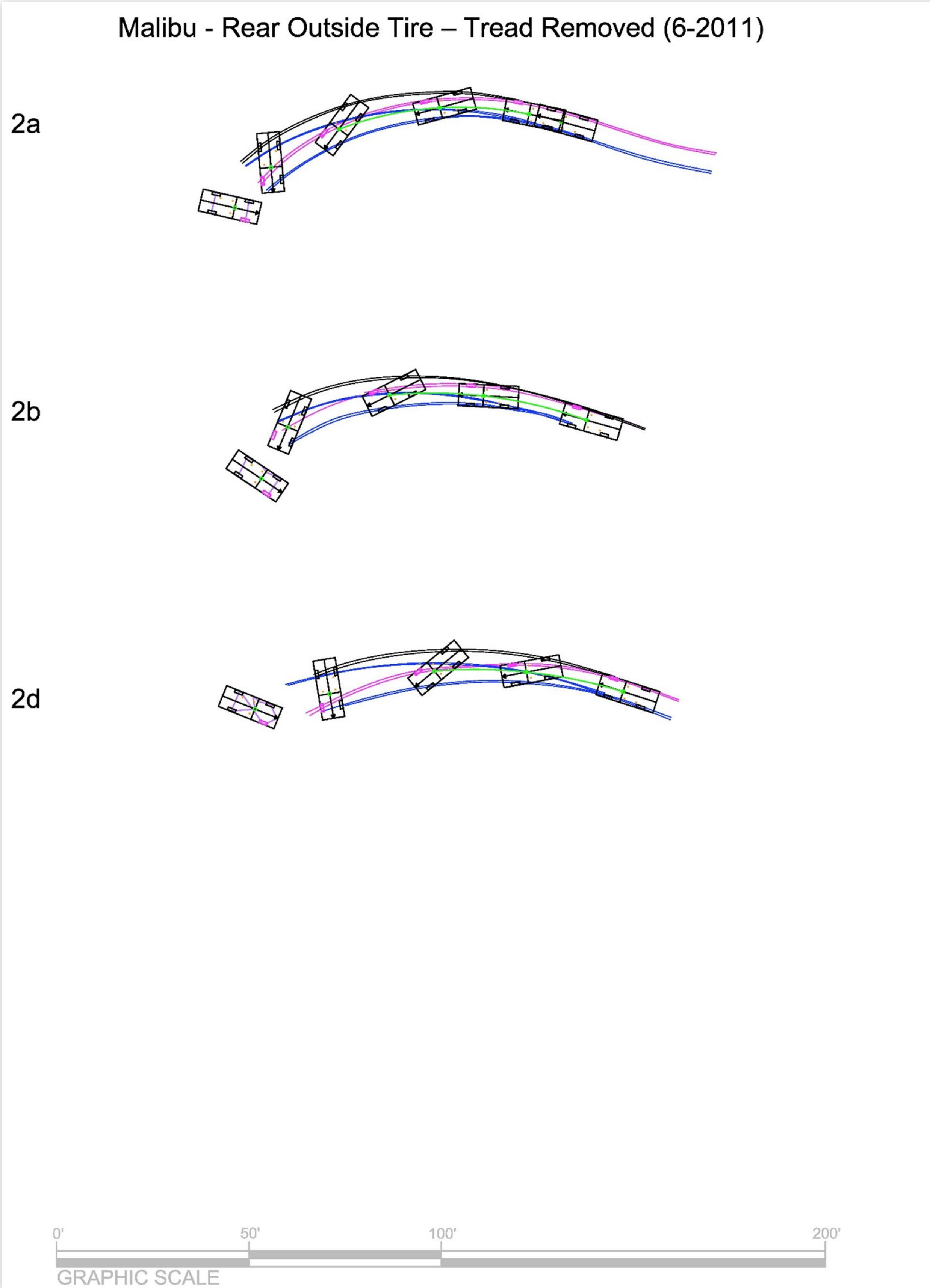
100'
200'
GRAPHIC SCALE

Appendix C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2b
2d
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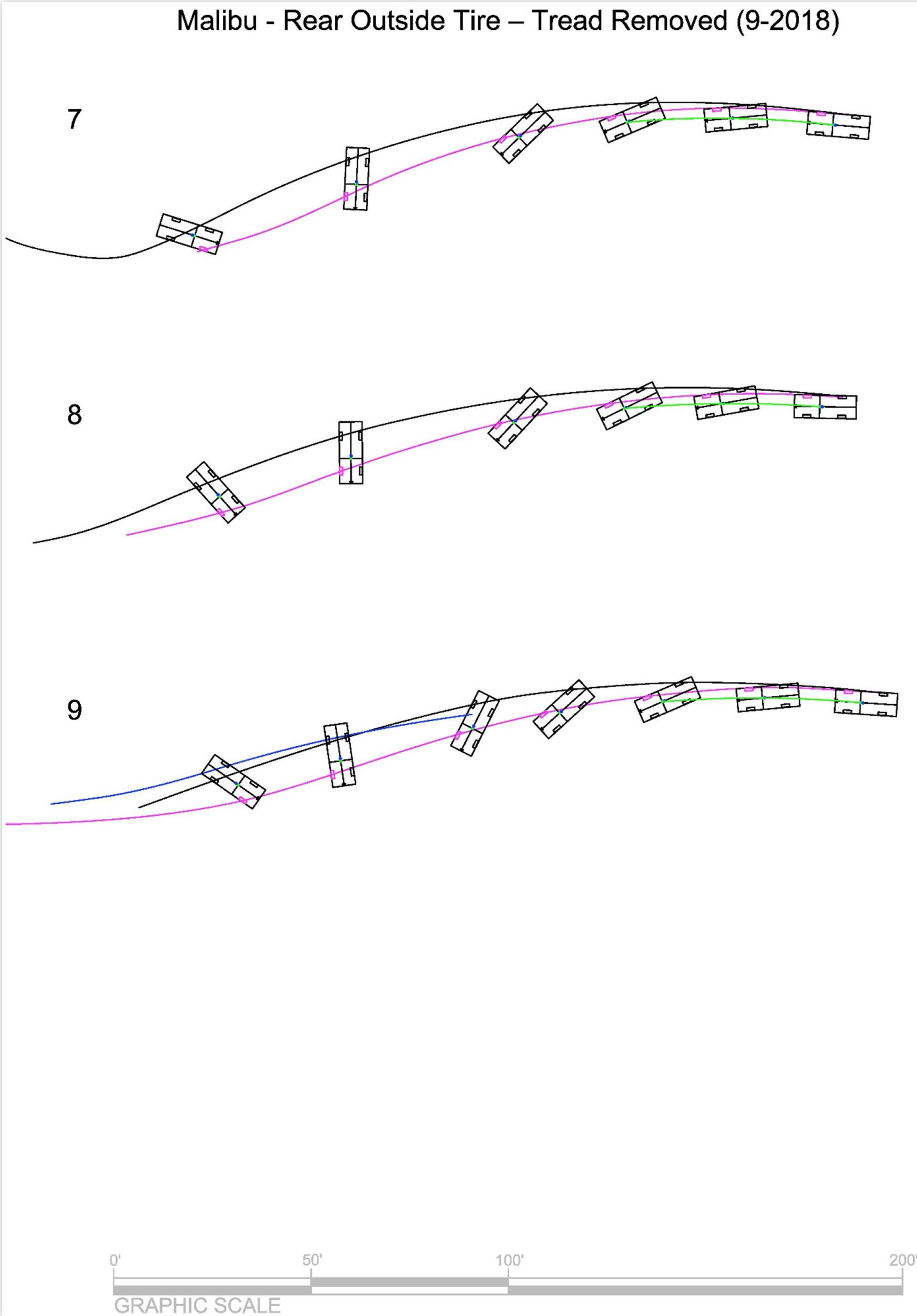
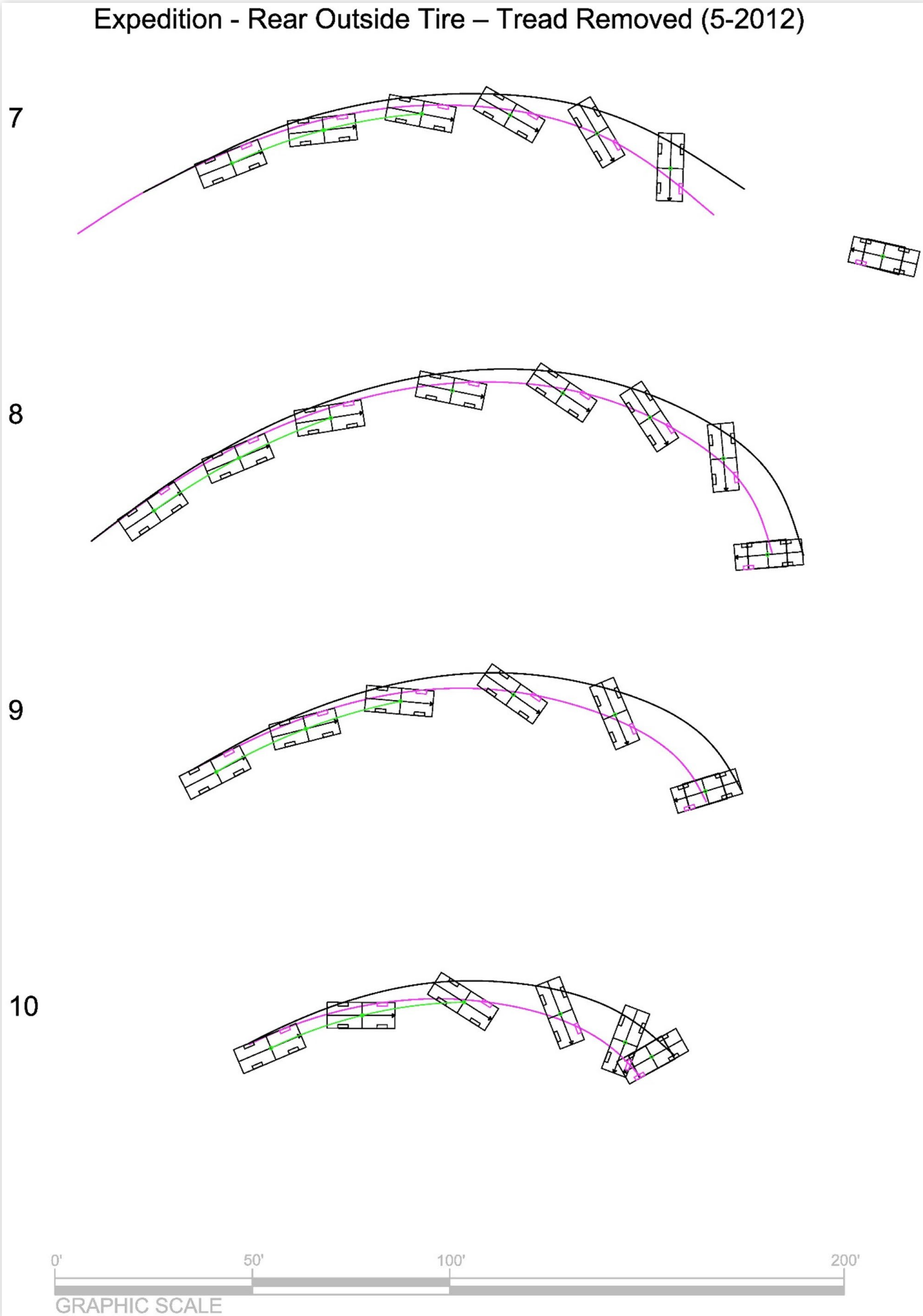
50'
100'
200'
GRAPHIC SCALE


Malibu - Rear Outside Tire – Tread Removed (9-2018)
7
8
9
0'
50'
100'
200'
GRAPHIC SCALE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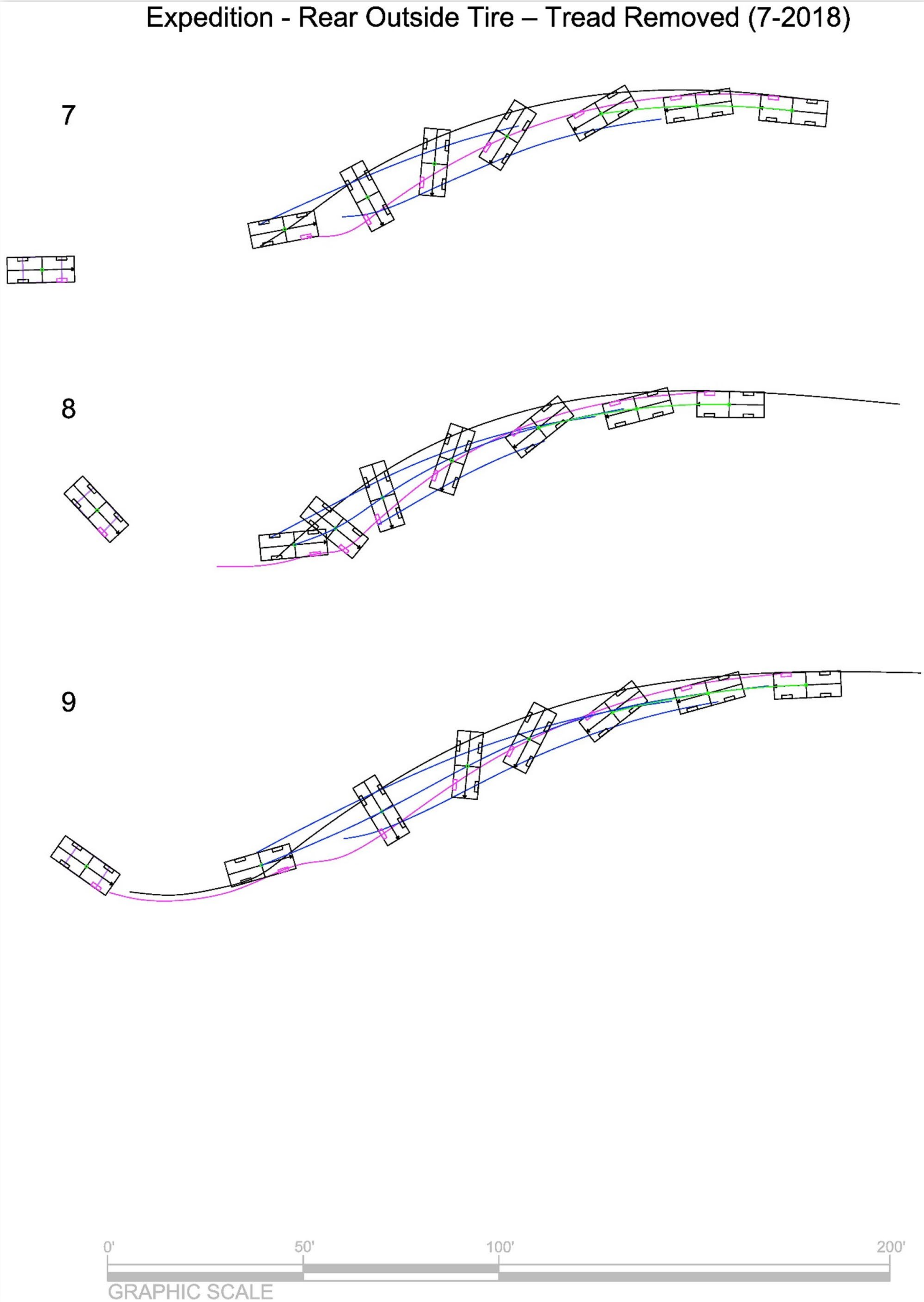
GRAPHIC SCALE


Expedition - Rear Outside Tire – Tread Removed (7-2018)
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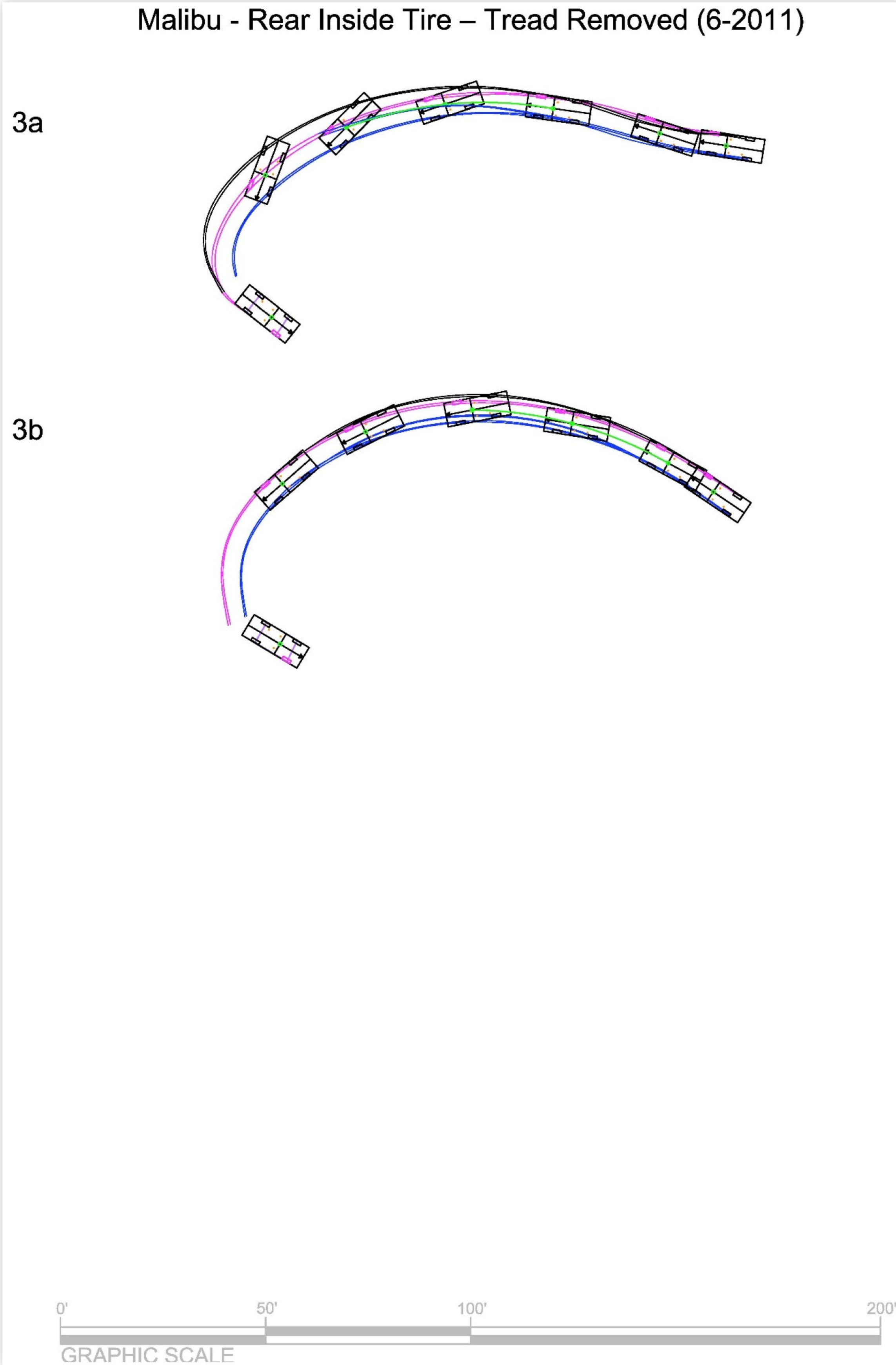
8
9
0'
50'
100'
200'
GRAPHIC SCALE

Appendix D


Malibu - Rear Inside Tire – Tread Removed (6-2011)
3a
3b
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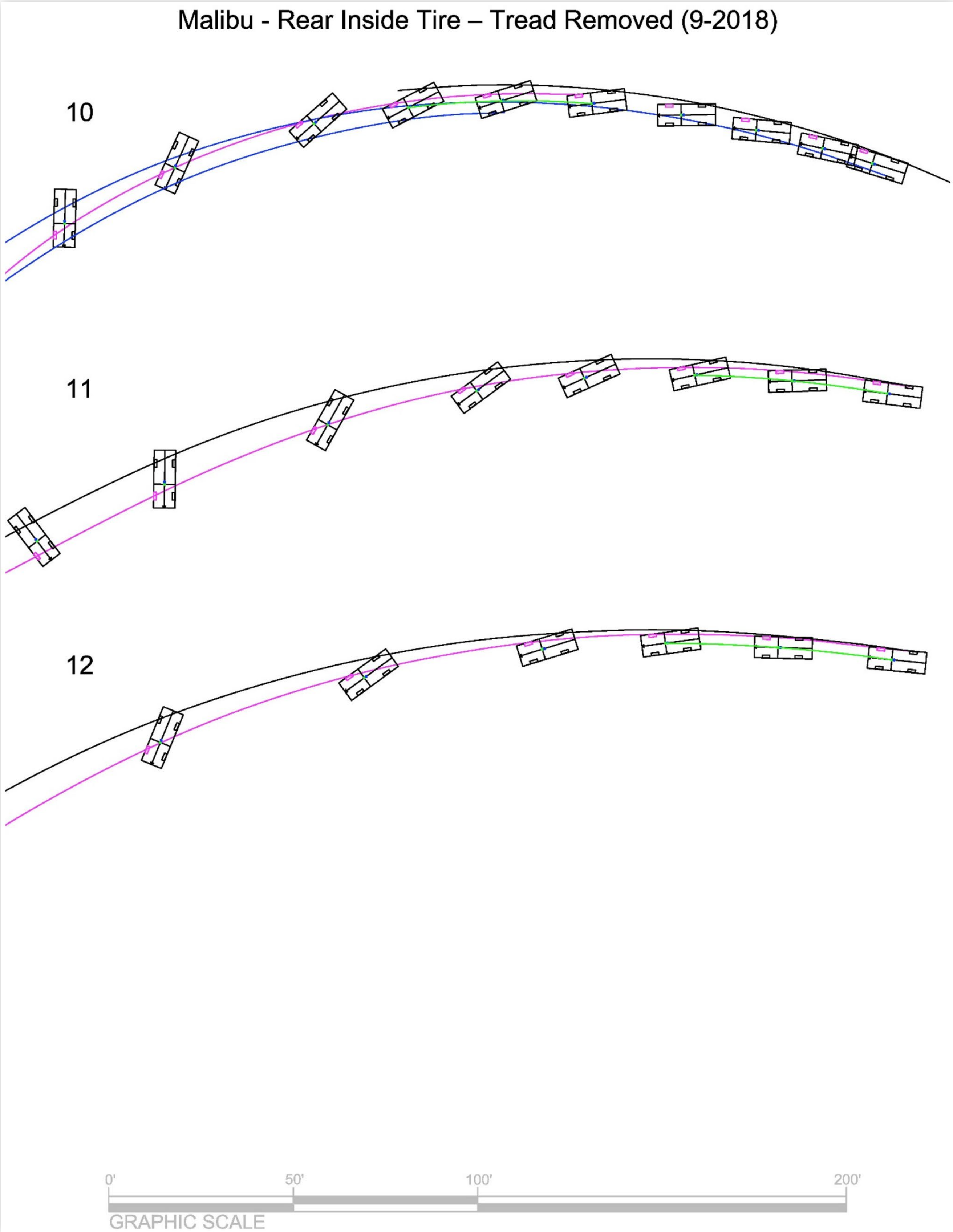
50'
100'
200'
GRAPHIC SCALE


Malibu - Rear Inside Tire – Tread Removed (9-2018)
10
11
12
0'
50'
100'
200'
GRAPHIC SCALE

Appendix E

Four Unaltered Tires (Front Outside Tire Measurement)			
	SQR Error	Min	Max
All Tires - Full Friction	23%	-11%	0%
Coasting Adjusted			
All Tires - Full Friction	16%	-7%	5%

Four Unaltered Tires (Center of Gravity Measurement)			
	SQR Error	Min	Max
All Tires - Full Friction	22%	-12%	0%
Coasting Adjusted			
All Tires - Full Friction	14%	-8%	5%

Front Outside Tire – Tread Removed
(Front Outside Tire Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	49%	14%	22%
Subject Tire - Reduced Friction	38%	10%	18%
Subject Axle - Reduced Friction	28%	6%	14%
All Tires - Reduced Friction	16%	-9%	5%
Coasting Adjusted			
All Tires - Full Friction	67%	19%	32%
Subject Tire - Reduced Friction	56%	15%	27%
Subject Axle - Reduced Friction	45%	11%	23%
All Tires - Reduced Friction	19%	-5%	10%

Front Outside Tire – Tread Removed
(Center of Gravity Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	52%	14%	32%
Subject Tire - Reduced Friction	42%	10%	27%
Subject Axle - Reduced Friction	32%	6%	23%
All Tires - Reduced Friction	12%	-8%	5%
Coasting Adjusted			
All Tires - Full Friction	67%	19%	38%
Subject Tire - Reduced Friction	56%	15%	33%
Subject Axle - Reduced Friction	45%	11%	28%
All Tires - Reduced Friction	16%	-4%	10%

Rear Outside Tire – Tread Removed

(Front Outside Tire Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	35%	-20%	6%
Subject Tire - Reduced Friction	44%	-23%	3%
Subject Axle - Reduced Friction	54%	-26%	-1%
All Tires - Reduced Friction	87%	-36%	-9%
Coasting Adjusted			
All Tires - Full Friction	26%	-17%	11%
Subject Tire - Reduced Friction	33%	-19%	7%
Subject Axle - Reduced Friction	42%	-22%	3%
All Tires - Reduced Friction	75%	-33%	-4%

Rear Outside Tire – Tread Removed

(Center of Gravity Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	20%	-11%	8%
Subject Tire - Reduced Friction	25%	-14%	4%
Subject Axle - Reduced Friction	34%	-17%	1%
All Tires - Reduced Friction	68%	-29%	-7%
Coasting Adjusted			
All Tires - Full Friction	25%	-6%	13%
Subject Tire - Reduced Friction	20%	-10%	9%
Subject Axle - Reduced Friction	23%	-13%	5%
All Tires - Reduced Friction	56%	-25%	-3%

Rear Inside Tire – Tread Removed

(Front Outside Tire Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	19%	-9%	-8%
Subject Tire - Reduced Friction	26%	-12%	-11%
Subject Axle - Reduced Friction	33%	-16%	-14%
All Tires - Reduced Friction	61%	-28%	-26%
Coasting Adjusted			
All Tires - Full Friction	10%	-5%	-3%
Subject Tire - Reduced Friction	17%	-8%	-7%
Subject Axle - Reduced Friction	25%	-12%	-10%
All Tires - Reduced Friction	53%	-24%	-23%

Rear Inside Tire – Tread Removed

(Center of Gravity Measurement)

	SQR Error	Min	Max
All Tires - Full Friction	17%	-10%	-2%
Subject Tire - Reduced Friction	24%	-13%	-5%
Subject Axle - Reduced Friction	31%	-16%	-8%
All Tires - Reduced Friction	58%	-28%	-22%
Coasting Adjusted			
All Tires - Full Friction	9%	-5%	3%
Subject Tire - Reduced Friction	15%	-9%	-1%
Subject Axle - Reduced Friction	22%	-12%	-4%
All Tires - Reduced Friction	50%	-25%	-18%

Appendix F

Malibu - Four Unaltered Tires (Center of Gravity Measurement)

Testing Date	6/24/2011				7/24/2018						9/28/2018					
Test Number	1 (1d)		2 (1f)		1		2		3		1		2		3	
Actual Speed (mph)	39.8		40.5		48.4		52.1		58.6		55.4		54.4		62.2	
Ending Slip Angle (deg)	5		4		15		9		19		15		24		22	
All Tires - Full Friction	35.0	-12%	36.3	-10%	48.4	0%	51.1	-2%	56.1	-4%	51.9	-6%	50.1	-8%	60.4	-3%
Coasting Adjusted																
All Tires - Full Friction	36.6	-8%	38.0	-6%	50.6	5%	53.5	3%	58.7	0%	54.3	-2%	52.4	-4%	63.2	2%

Malibu - Front Outside Tire - Tread Removed (Center of Gravity Measurement)

Testing Date	9/28/2018					
Test Number	4		5		6	
Actual Speed (mph)	61.3		56.6		58.3	
Ending Slip Angle (deg)	2		3		1	
All Tires - Full Friction	72.3	18%	74.5	32%	67.1	15%
Subject Tire - Reduced Friction	69.9	14%	72.0	27%	64.8	11%
Subject Axle - Reduced Friction	67.4	10%	69.5	23%	62.5	7%
All Tires - Reduced Friction	57.7	-6%	59.5	5%	53.5	-8%
Coasting Adjusted						
All Tires - Full Friction	75.6	23%	77.9	38%	70.1	20%
Subject Tire - Reduced Friction	73.1	19%	75.4	33%	67.8	16%
Subject Axle - Reduced Friction	70.5	15%	72.7	28%	65.4	12%
All Tires - Reduced Friction	60.4	-2%	62.2	10%	56.0	-4%

Malibu - Rear Outside Tire – Tread Removed (Center of Gravity Measurement)

Testing Date	6/24/2011						9/28/2018					
Test Number	3 (2a)		4 (2b)		5 (2d)		7		8		9	
Actual Speed (mph)	33.4		36.7		37.5		57.1		64.5		65.2	
Ending Slip Angle (deg)	33		21		35		18		21		19	
All Tires - Full Friction	31.0	-7%	35.6	-3%	37.9	1%	54.6	-4%	60.1	-7%	58.3	-11%
Subject Tire - Reduced Friction	29.9	-10%	34.3	-7%	36.5	-3%	52.8	-8%	58.1	-10%	56.4	-14%
Subject Axle - Reduced Friction	28.8	-14%	33.0	-10%	35.1	-6%	50.9	-11%	56.0	-13%	54.4	-17%
All Tires - Reduced Friction	24.7	-26%	28.3	-23%	30.2	-20%	43.6	-24%	48.0	-26%	46.5	-29%
Coasting Adjusted												
All Tires - Full Friction	32.5	-3%	37.2	1%	39.6	6%	57.1	0%	62.9	-3%	61.0	-6%
Subject Tire - Reduced Friction	31.3	-6%	35.9	-2%	38.2	2%	55.2	-3%	60.8	-6%	59.0	-10%
Subject Axle - Reduced Friction	30.1	-10%	34.5	-6%	36.7	-2%	53.2	-7%	58.6	-9%	56.9	-13%
All Tires - Reduced Friction	25.8	-23%	29.6	-19%	31.5	-16%	45.6	-20%	50.2	-22%	48.7	-25%

Malibu - Rear Inside Tire – Tread Removed (Center of Gravity Measurement)

Testing Date	6/24/2011				9/28/2018					
Test Number	6 (3a)		7 (3b)		10		11		12	
Actual Speed (mph)	35.0		36.1		59.1		68.8		72.8	
Ending Slip Angle (deg)	25		12		21		14		8	
All Tires - Full Friction	32.3	-8%	32.6	-10%	53.7	-9%	64.1	-7%	71.6	-2%
Subject Tire - Reduced Friction	31.1	-11%	31.5	-13%	51.9	-12%	62.0	-10%	69.2	-5%
Subject Axle - Reduced Friction	29.9	-15%	30.3	-16%	50.1	-15%	59.8	-13%	66.8	-8%
All Tires - Reduced Friction	25.7	-27%	26.0	-28%	42.9	-27%	51.2	-26%	57.1	-22%
Coasting Adjusted										
All Tires - Full Friction	33.8	-4%	34.1	-5%	56.2	-5%	67.1	-3%	74.9	3%
Subject Tire - Reduced Friction	32.6	-7%	32.9	-9%	54.3	-8%	64.8	-6%	72.4	-1%
Subject Axle - Reduced Friction	31.3	-11%	31.7	-12%	52.4	-11%	62.5	-9%	69.8	-4%
All Tires - Reduced Friction	26.9	-23%	27.2	-25%	44.8	-24%	53.5	-22%	59.8	-18%

Malibu - Four Unaltered Tires (Front Outside Tire Measurement)

Testing Date	6/24/2011				7/24/2018						9/28/2018					
Test Number	1 (1d)		2 (1f)		1		2		3		1		2		3	
Actual Speed (mph)	39.8		40.5		48.4		52.1		58.6		55.4		54.4		62.2	
Ending Slip Angle (deg)	5		4		15		9		19		15		24		22	
All Tires - Full Friction	35.6	-10%	36.6	-10%	47.5	-2%	50.0	-4%	57.1	-2%	49.5	-11%	50.7	-7%	57.1	-8%
Coasting Adjusted																
All Tires - Full Friction	37.3	-6%	38.3	-5%	49.7	3%	52.3	0%	59.8	2%	51.8	-6%	50.7	-7%	59.7	-4%

Malibu - Front Outside Tire – Tread Removed (Front Outside Tire Measurement)

Testing Date	9/28/2018					
Test Number	4		5		6	
Actual Speed (mph)	61.3		56.6		58.3	
Ending Slip Angle (deg)	2		3		1	
All Tires - Full Friction	70.3	15%	66.3	17%	66.3	14%
Subject Tire - Reduced Friction	68.0	11%	64.2	13%	64.2	10%
Subject Axle - Reduced Friction	65.6	7%	61.9	9%	61.9	6%
All Tires - Reduced Friction	56.1	-8%	53.0	-6%	53.0	-9%
Coasting Adjusted						
All Tires - Full Friction	73.6	20%	74.6	32%	69.4	19%
Subject Tire - Reduced Friction	71.1	16%	72.1	27%	67.1	15%
Subject Axle - Reduced Friction	68.6	12%	69.6	23%	64.7	11%
All Tires - Reduced Friction	58.7	-4%	59.5	5%	55.4	-5%

Malibu - Rear Outside Tire – Tread Removed (Front Outside Tire Measurement)

Testing Date	6/24/2011						9/28/2018					
Test Number	3 (2a)		4 (2b)		5 (2d)		7		8		9	
Actual Speed (mph)	33.4		36.7		37.5		57.1		64.5		65.2	
Ending Slip Angle (deg)	33		21		35		18		21		19	
All Tires - Full Friction	30.2	-10%	32.8	-11%	36.9	-2%	50.3	-12%	55.4	-14%	52.0	-20%
Subject Tire - Reduced Friction	29.1	-13%	31.7	-14%	35.5	-5%	48.7	-15%	53.5	-17%	50.3	-23%
Subject Axle - Reduced Friction	28.0	-16%	30.4	-17%	34.2	-9%	46.9	-18%	51.7	-20%	48.5	-26%
All Tires - Reduced Friction	24.0	-28%	26.1	-29%	29.3	-22%	40.2	-30%	44.2	-31%	41.5	-36%
Coasting Adjusted												
All Tires - Full Friction	31.5	-6%	34.3	-6%	38.6	3%	52.6	-8%	57.9	-10%	54.4	-17%
Subject Tire - Reduced Friction	30.4	-9%	33.1	-10%	37.2	-1%	50.9	-11%	56.0	-13%	52.6	-19%
Subject Axle - Reduced Friction	29.3	-12%	31.8	-13%	35.8	-5%	49.1	-14%	54.0	-16%	50.7	-22%
All Tires - Reduced Friction	25.1	-25%	27.3	-26%	30.7	-18%	42.0	-26%	46.2	-28%	43.4	-33%

Malibu - Rear Inside Tire – Tread Removed (Front Outside Tire Measurement)

Testing Date	6/24/2011				9/28/2018					
Test Number	6 (3a)		7 (3b)		10		11		12	
Actual Speed (mph)	35.0		36.1		59.1		68.8		72.8	
Ending Slip Angle (deg)	25		12		21		14		8	
All Tires - Full Friction	32.2	-8%	32.8	-9%	54.0	-9%	62.4	-9%	67.2	-8%
Subject Tire - Reduced Friction	31.1	-11%	31.6	-12%	52.2	-12%	60.4	-12%	64.9	-11%
Subject Axle - Reduced Friction	29.9	-15%	30.4	-16%	50.4	-15%	58.2	-15%	62.6	-14%
All Tires - Reduced Friction	25.6	-27%	26.1	-28%	43.1	-27%	49.8	-28%	53.6	-26%
Coasting Adjusted										
All Tires - Full Friction	33.7	-4%	34.3	-5%	56.5	-4%	65.3	-5%	70.3	-3%
Subject Tire - Reduced Friction	32.5	-7%	33.1	-8%	54.6	-8%	63.1	-8%	67.9	-7%
Subject Axle - Reduced Friction	31.2	-11%	31.8	-12%	52.7	-11%	60.9	-11%	65.5	-10%
All Tires - Reduced Friction	26.8	-23%	27.3	-24%	45.1	-24%	52.1	-24%	56.1	-23%

Expedition - Four Unaltered Tires (Center of Gravity Measurement)

Testing Date	5/31/2012						7/24/2018					
Test Number	5a		5b		5c		1		2		3	
Actual Speed (mph)	44.6		41.2		42.0		46.2		46.8		47.1	
Ending Slip Angle (deg)	0		4		11		9		8		10	
All Tires - Full Friction	58.9	35%	40.2	-2%	39.7	-5%	45.9	-1%	45.6	-3%	44.1	-6%
Coasting Adjusted												
All Tires - Full Friction	61.6	41%	42.1	2%	41.6	-1%	48.0	4%	47.7	2%	46.1	-2%

Expedition - Front Outside Tire – Tread Removed (Center of Gravity Measurement)

Testing Date	5/31/2012						7/24/2018					
Test Number	6a		6b		6c		4		5		6	
Actual Speed (mph)	41.3		45.0		45.3		48.4		48.9		51.6	
Ending Slip Angle (deg)	0		0		6		3		3		2	
All Tires - Full Friction	72.9	77%	69.7	55%	54.1	19%	56.3	16%	58.2	19%	58.7	14%
Subject Tire - Reduced Friction	70.6	71%	67.4	50%	52.3	16%	54.4	12%	56.3	15%	56.8	10%
Subject Axle - Reduced Friction	68.1	65%	65.1	45%	50.5	12%	52.5	8%	54.3	11%	54.8	6%
All Tires - Reduced Friction	62.8	52%	60.0	33%	46.6	3%	48.4	0%	50.0	2%	50.5	-2%
Coasting Adjusted												
All Tires - Full Friction	76.3	85%	72.9	62%	56.6	25%	58.9	22%	60.8	24%	61.4	19%
Subject Tire - Reduced Friction	73.8	79%	70.5	57%	54.7	21%	56.9	18%	58.9	20%	59.4	15%
Subject Axle - Reduced Friction	71.3	73%	68.1	51%	52.8	17%	54.9	13%	56.8	16%	57.3	11%
All Tires - Reduced Friction	65.7	59%	62.7	39%	48.7	7%	50.6	5%	52.3	7%	52.8	2%

Expedition - Rear Outside Tire – Tread Removed (Center of Gravity Measurement)

Testing Date	5/31/2012								7/24/2018					
Test Number	7		8		9		10		7		8		9	
Actual Speed (mph)	40.4		41.2		39.4		37.6		44.4		42.0		47.7	
Ending Slip Angle (deg)	15		10		17		34		17		24		24	
All Tires - Full Friction	38.9	-4%	44.5	8%	41.2	5%	36.3	-3%	45.9	3%	43.7	4%	50.5	6%
Subject Tire - Reduced Friction	37.6	-7%	43.0	4%	39.8	1%	35.1	-7%	44.4	0%	42.2	1%	48.8	2%
Subject Axle - Reduced Friction	36.2	-10%	41.4	1%	38.4	-3%	33.8	-10%	42.9	-3%	40.7	-3%	47.1	-1%
All Tires - Reduced Friction	33.5	-17%	38.3	-7%	35.5	-10%	31.2	-17%	39.5	-11%	37.5	-11%	43.4	-9%
Coasting Adjusted														
All Tires - Full Friction	40.7	1%	46.5	13%	43.1	9%	38.0	1%	48.1	8%	45.7	9%	52.8	11%
Subject Tire - Reduced Friction	39.3	-3%	45.0	9%	41.7	6%	36.7	-2%	46.5	5%	44.2	5%	51.1	7%
Subject Axle - Reduced Friction	37.9	-6%	43.3	5%	40.2	2%	35.4	-6%	44.9	1%	42.6	1%	49.3	3%
All Tires - Reduced Friction	35.0	-13%	40.0	-3%	37.1	-6%	32.7	-13%	41.3	-7%	39.3	-7%	45.4	-5%

Expedition - Four Unaltered Tires (Front Outside Tire Measurement)

Testing Date	5/31/2012						7/24/2018					
Test Number	5a		5b		5c		1		2		3	
Actual Speed (mph)	44.6		41.2		42.0		46.2		46.8		47.1	
Ending Slip Angle (deg)	0		4		11		9		8		10	
All Tires - Full Friction	56	26%	41.3	0%	39.8	-5%	46.0	0%	46.9	0%	43.9	-7%
Coasting Adjusted												
All Tires - Full Friction	59	32%	43.2	5%	41.7	-1%	48.2	4%	49.1	5%	46.0	-2%

Expedition - Front Outside Tire – Tread Removed (Front Outside Tire Measurement)

Testing Date	5/31/2012						7/24/2018					
Test Number	6a		6b		6c		4		5		6	
Actual Speed (mph)	41.3		45.0		45.3		48.4		48.9		51.6	
Ending Slip Angle (deg)	0		0		6		3		3		2	
All Tires - Full Friction	67.4	63%	67.7	50%	55.4	22%	56.2	16%	59.6	22%	62.1	20%
Subject Tire - Reduced Friction	65.3	58%	65.5	46%	53.6	18%	54.3	12%	57.6	18%	60.0	16%
Subject Axle - Reduced Friction	63.0	53%	63.3	41%	51.8	14%	52.4	8%	55.6	14%	57.9	12%
All Tires - Reduced Friction	58.0	41%	58.3	30%	47.7	5%	48.3	0%	51.2	5%	53.4	3%
Coasting Adjusted												
All Tires - Full Friction	70.5	71%	70.8	57%	58.0	28%	58.8	21%	62.3	27%	64.9	26%
Subject Tire - Reduced Friction	68.3	65%	68.5	52%	56.1	24%	56.8	17%	60.3	23%	62.8	22%
Subject Axle - Reduced Friction	65.9	60%	66.2	47%	54.2	20%	54.9	13%	58.2	19%	60.6	17%
All Tires - Reduced Friction	60.7	47%	61.0	36%	49.9	10%	50.5	4%	53.6	10%	55.8	8%

Expedition - Rear Outside Tire – Tread Removed (Front Outside Tire Measurement)

Testing Date	5/31/2012								7/24/2018					
Test Number	7		8		9		10		7		8		9	
Actual Speed (mph)	40.4		41.2		39.4		37.6		44.4		42.0		47.7	
Ending Slip Angle (deg)	15		10		17		34		17		24		24	
All Tires - Full Friction	38.7	-4%	43.7	6%	38.8	-1%	34.1	-9%	41.4	-7%	40.0	-5%	44.0	-8%
Subject Tire - Reduced Friction	37.4	-7%	42.3	3%	37.5	-5%	32.9	-12%	40.0	-10%	38.7	-8%	42.5	-11%
Subject Axle - Reduced Friction	36.0	-11%	40.7	-1%	36.2	-8%	31.7	-16%	38.6	-13%	37.3	-11%	41.0	-14%
All Tires - Reduced Friction	33.3	-18%	37.6	-9%	33.4	-15%	29.3	-22%	35.6	-20%	34.4	-18%	37.8	-21%
Coasting Adjusted														
All Tires - Full Friction	40.5	0%	45.7	11%	40.6	3%	35.6	-5%	43.3	-3%	41.8	0%	46.0	-4%
Subject Tire - Reduced Friction	39.1	-3%	44.2	7%	39.3	0%	34.4	-8%	41.9	-6%	40.5	-4%	44.5	-7%
Subject Axle - Reduced Friction	37.7	-7%	42.6	3%	37.8	-4%	33.2	-12%	40.4	-9%	39.0	-7%	42.9	-10%
All Tires - Reduced Friction	34.8	-14%	39.4	-4%	35.0	-11%	30.7	-18%	37.2	-16%	36.0	-14%	39.5	-17%

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